

A Stochastic Frontier Model with  
Time-Varying Vectorautoregressive Inefficiency\*  
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**Abstract**

In this paper we propose an extension to the approach suggested by Tsionas (2006) to consider time delays in the adjustment process of efficiency. We allow the dynamic adjustments towards the technological frontier to follow a vector autoregressive model with time varying parameters. Focussing on the two main regional aggregates for Italy - Centre-North and Mezzogiorno - over the observed period 1979-2003, we apply this method to identify the channels through which core and non-core public infrastructures affect regional differences in per capita GDP.

**JEL Codes:** D24 O47 C13 C33 C39 E32

**key words:** Time-varying VAR, Technical Efficiency,  
Stochastic Frontier, Panel Data, Public Investment.

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# 1 Introduction

It is well known that the standard Solow (1957) residual can only be interpreted as a measure for technical progress under very strong assumptions. Controlling for non constant returns to scale and the presence of market power (Hall 1990), or cyclical factor utilization (Basu 1996, Basu and Kimball 1997, Basu and Fernald 2001), leads to different dynamics over the business cycle. These results have important implications for the role of technical progress in business cycle models.<sup>1</sup>

Pioneered by Aigner et al. (1977) and Meeusen and van den Broeck (1977), stochastic frontier analysis is a possibility to address this issue, allowing for changes in factor utilization. Consider a production frontier of Cobb-Douglas type, with two inputs (capital  $K_{j,t}$  and labor  $L_{j,t}$ ) and an output ( $Y_{j,t}$ ). There are  $j = 1, \dots, N$  units of production. Efficiency is measured as  $\tau_{j,t}$ ,  $0 < \tau_{j,t} \leq 1$ ,<sup>2</sup> and  $\omega_{j,t}$  captures the stochastic nature of the frontier. In log-linear form, the model is

$$y_{j,t} = \alpha l_{j,t} + \beta k_{j,t} + v_{j,t} - u_{j,t}, \quad (1)$$

where  $u_{j,t} = -\ln \tau_{j,t}$  is a non-negative variable, and  $v_{j,t} = \ln \omega_{j,t}$  is a normally distributed error with mean zero. Such a model can be estimated using the maximum likelihood approach (e.g., Battese and Coelli 1995) or Bayesian techniques (e.g., Koop et al. 1999), under the assumption that the  $u_{j,t}$  are independent. This assumption has been relaxed recently by Tsionas (2006), who models log inefficiency as following an AR process.

The business cycle phenomenon is not stable over time. A recent example of changes in the characteristics of business cycles is the “Great Moderation”, the decline in volatility of GDP growth since the 1980s.<sup>3</sup> Interpreting inefficiency as capturing cyclical factor variation seems to require allowing for

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<sup>1</sup>See, e.g. Galí (1999), Basu et al. (1998), and Malley et al. (2005).

<sup>2</sup>When  $\tau = 1$  there is full efficiency, in this case the unit  $j$  produces on the efficiency frontier.

<sup>3</sup>This decline was first noted by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) for the volatility of US GDP growth, but can also be found in other industrialized countries (Summers 2005).

changes in business cycle patterns.

Recent work by Cogley and Sargent (2005) and Primiceri (2005) demonstrates how to allow for time varying parameters and stochastic volatility in monetary policy analysis using vector autoregressive models. In our paper, we extend the model suggested by Tsionas (2006) along the lines of Primiceri (2005): log-inefficiency follows a VAR with time dependent parameters. In addition, the variance-covariance matrix of inefficiency innovations can vary over time.

We apply this method to extend the analysis of different long-term growth paths in the Italian regions in Mastromarco and Woitek (2006), discriminating between the channels through which public infrastructure influences overall productivity. Since there are changes in the pattern of regional business cycles in Italy (Mastromarco and Woitek 2007), it seems natural to allow these changes to affect cyclical factor utilization in the regional production functions. The paper is structured as follows: the data are described in Section 2 and Section 3 introduces the method. Section 4 discusses the results, and Section 5 concludes.

## 2 Data

We analyse annual data for 20 Italian regions.<sup>4</sup> Due to data availability, the observation period is restricted to 1970 to 1994. Output, capital stock and labour input are from CRENOS NewRegioIt60-96. The output measure is regional gross domestic product at constant 1985 prices, the capital stock is also at constant 1985 prices.<sup>5</sup> The measure for labor input is the number of workers. Workers in different regions may have different levels of skill, due

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<sup>4</sup>Center-North (CEN): PIE: Piemonte; VDA: Valle D'Aosta; LOM: Lombardia; TAA: Trentino/Alto Adige; VEN: Veneto; FVG: Friuli/Venezia/Giulia; LIG: Liguria; EMR: Emilia Romagna; TOS: Toscana; UMB: Umbria; MAR: Marche; LAZ: Lazio; Mezzogiorno (MEZ): ABR: Abruzzo; MOL: Molise; CAM: Campania; PUG: Puglia; BAS: Basilicata; CAL: Calabria; SIC: Sicilia; SAR: Sardegna. We will also use the following breakdown: North-West: PIE, VDA, LOM, LIG; North-East: TAA, VEN, FVG, EMR; Center: TOS, UMB, MAR, LAZ; South-East: ABR, MOL, PUG; South-West: CAM, BAS, CAL, SIC, SAR.

<sup>5</sup>The deflators are calculated at the regional level (for details, see Paci and Saba 1998).

to differences in education and experience. Following Benhabib and Spiegel (1994), Tallman and Wang (1994) and Koop et al. (2000), we treat education as labor enhancing. Given the lack of data on average years of attainment at the secondary level, we use number of people registered in high school.<sup>6</sup> Public capital stocks (constant 1985 prices) is from Picci (1999) and Bonaglia and Picci (2000) and is decomposed into streets and airports, trains and subways, ports, electrical lines and water, telecommunications, public buildings, hospitals, reclaimed land and others.<sup>7</sup> We follow Picci (1999) and calculate two aggregates: core infrastructure (transport, electricity, communication) and non-core infrastructure (all the rest). In the following, all variables are in logs.

### 3 Method

The model is an extension to Tsionas (2006), allowing for log inefficiency following a VAR of order  $p$  and time varying parameters, using Cogley and Sargent (2005) and Primiceri (2005).

Consider  $N$  units producing a vector of outputs  $\mathbf{y}_t$  at time  $t$ . The production function is specified as Cobb-Douglas, with a time trend in total factor productivity:

$$\mathbf{y}_t = \mathbf{x}_t \boldsymbol{\beta} + \mathbf{v}_t - \mathbf{u}_t; \mathbf{v}_t \sim N(\mathbf{0}, \boldsymbol{\Omega}_v), \quad (2)$$

with

$$\mathbf{x}_t = \begin{pmatrix} 1 & t & l_{1,t} & k_{1,t} \\ 1 & t & l_{2,t} & k_{2,t} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t & l_{N,t} & k_{N,t} \end{pmatrix}.$$

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<sup>6</sup>This is done by multiplying the original series by an index measuring the proportion of population registered at high school. Enrollment ratios have been used as proxies for human capital in the seminal studies of Romer (1990), Barro (1991) and Mankiw et al. (1992) and in the sensitivity study by Levine and Renelt (1992), among many others. As shown by Barro and Sala-i-Martin (1995, p.437), using attainment variables instead does not lead to qualitatively different impacts on growth, which makes us more confident about the results presented here.

<sup>7</sup>The data can be downloaded from <http://www.spbo.unibo.it/picci/indexkp.html>.

We allow the measurement errors for each unit to have a different variance, and to be correlated with each other. The parameter vector  $\boldsymbol{\beta}$  has to fulfill the regularity condition that capital and labor elasticities are non-negative. Stacking equation (2) gives

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{v} - \mathbf{u}, \mathbf{y} = \begin{pmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_T \end{pmatrix}; \mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_T \end{pmatrix}; \mathbf{v} = \begin{pmatrix} \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_T \end{pmatrix}; \mathbf{u} = \begin{pmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_T \end{pmatrix}. \quad (2')$$

Given  $\mathbf{u}_{1,\dots,T}$ , the likelihood of the model in (2') is

$$f(\mathbf{y}_{1,\dots,T}|\boldsymbol{\beta}, \boldsymbol{\Omega}, \mathbf{u}_{1,\dots,T}) \propto |\boldsymbol{\Omega}|^{-\frac{NT}{2}} \exp\left(-\frac{1}{2} \sum_{t=1}^T (\mathbf{y}_t + \mathbf{u}_t - \mathbf{X}_t\boldsymbol{\beta})' \boldsymbol{\Omega}^{-1} (\mathbf{y}_t + \mathbf{u}_t - \mathbf{X}_t\boldsymbol{\beta})\right), \quad (3)$$

The prior for  $\boldsymbol{\Omega}_v$  is inverted Wishart,  $p(\boldsymbol{\Omega}_v) = IW(\nu_0, \mathbf{V}_0)$ , and the prior for the parameter vector  $\boldsymbol{\beta}$  is truncated normal  $p(\boldsymbol{\beta}) = N(\boldsymbol{\beta}_0, \mathbf{B}_0)I(\boldsymbol{\beta})$ , where  $I(\cdot)$  is an indicator function taking the value 1 if the vector  $\boldsymbol{\beta}$  fulfills the regularity conditions (non-negative elasticities) and 0 else. As conditional posterior, one obtains

$$\begin{aligned} \boldsymbol{\beta}|\mathbf{y}_{1,\dots,T}, \boldsymbol{\Omega} &\sim N(\boldsymbol{\beta}_1, \mathbf{B}_1)I(\boldsymbol{\beta}); \\ \boldsymbol{\beta}_1 &= \mathbf{B}_1 \left( \sum_{t=1}^T \mathbf{X}_t' \boldsymbol{\Omega}^{-1} (\mathbf{y}_t + \mathbf{u}_t) + \mathbf{B}_0^{-1} \boldsymbol{\beta}_0 \right); \\ \mathbf{B}_1 &= \left( \sum_{t=1}^T \mathbf{X}_t' \boldsymbol{\Omega}^{-1} \mathbf{X}_t + \mathbf{B}_0^{-1} \right)^{-1}. \end{aligned} \quad (4)$$

The variance-covariance matrix of the measurement error can be drawn from

an inverted Wishart distribution:

$$\begin{aligned}
\boldsymbol{\Omega}_v | \mathbf{y}_{1,\dots,T}, \boldsymbol{\beta}, \mathbf{u}_{1,\dots,T} &\sim IW(\nu_1, \mathbf{V}_1); \\
\nu_1 &= T + \nu_0; \\
\mathbf{V}_1 &= \left( \sum_{t=1}^T (\mathbf{y}_t + \mathbf{u}_t - \mathbf{X}_t \boldsymbol{\beta}) (\mathbf{y}_t + \mathbf{u}_t - \mathbf{X}_t \boldsymbol{\beta})' + \mathbf{V}_0 \right).
\end{aligned} \tag{5}$$

To allow for a rich dynamic structure in systematic deviations from the frontier reflecting e.g. cyclical factor utilization over the business cycle, we extend Battese and Coelli (1995), Kumbhakar et al. (1991), and Tsionas (2006), and model log inefficiency as vector autoregressive model with exogenous variables and time-varying parameters:<sup>8</sup>

$$\begin{aligned}
\tilde{\mathbf{u}}_t &= \ln(\mathbf{u}_t) = \\
&= \mathbf{z}_t \boldsymbol{\gamma}_t + \sum_{j=1}^p \mathbf{A}_{j,t} \tilde{\mathbf{u}}_{t-j} + \boldsymbol{\zeta}_t = \\
&= \mathbf{z}_t \boldsymbol{\gamma}_t + \begin{pmatrix} \mathbf{A}_{1,t} & \dots & \mathbf{A}_{p,t} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{u}}_{t-1} \\ \tilde{\mathbf{u}}_{t-2} \\ \vdots \\ \tilde{\mathbf{u}}_{t-p} \end{pmatrix} + \boldsymbol{\zeta}_t = \\
&= \mathbf{z}_t \boldsymbol{\gamma}_t + \mathbf{A}_t \tilde{\mathbf{U}}_{t-1} + \boldsymbol{\zeta}_t = \mathbf{z}_t \boldsymbol{\gamma}_t + \left( \tilde{\mathbf{U}}'_{t-1} \otimes \mathbf{I}_n \right) \boldsymbol{\alpha}_t + \boldsymbol{\zeta}_t = \\
&= \begin{pmatrix} \mathbf{z}_t & \left( \tilde{\mathbf{U}}'_{t-1} \otimes \mathbf{I} \right) \end{pmatrix} \begin{pmatrix} \boldsymbol{\gamma}_t \\ \boldsymbol{\alpha}_t \end{pmatrix} + \boldsymbol{\zeta}_t = \\
&= \mathbf{Z}_t \tilde{\boldsymbol{\alpha}}_t + \boldsymbol{\zeta}_t; \boldsymbol{\zeta}_t \sim N(\mathbf{0}, \boldsymbol{\Omega}_\zeta).
\end{aligned} \tag{6}$$

For the time paths of the paramters we assume random walks:

$$\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} + \boldsymbol{\nu}_{\alpha,t}, \tag{7}$$

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<sup>8</sup>Other than in Cogley and Sargent (2005) and Primiceri (2005), we assume homoskedasticity for the inefficiency errors to specify the model as parsimonious as possible. If the number of observations allows it, the extension to the heteroskedastic case is straightforward.

with

$$\boldsymbol{\nu}_{\alpha,t} \sim N(\mathbf{0}, \mathbf{V}).$$

Given the sequence  $\tilde{\mathbf{u}}_t$ , it is straightforward to draw from the distributions of the parameters and variance components:

1. Generate  $\boldsymbol{\alpha}_t$  and  $\boldsymbol{\gamma}_t, t = 1, \dots, T$  conditional on  $\mathbf{V}$  using Carter and Kohn (1994). Draws for which the resulting VAR is not stationary are rejected.
2. Generate  $\mathbf{V}$  conditional on  $\tilde{\boldsymbol{\alpha}}_t, t = 1, \dots, T$  from an inverted Wishart distribution.

To find the marginal posterior for the inefficiency terms  $\mathbf{u}_t$ , rewrite the system in (6) as VAR(1):<sup>9</sup>

$$\begin{aligned} \tilde{\mathbf{u}}_t &= \mathbf{z}_t \boldsymbol{\gamma}_t + \mathbf{A}_t \tilde{\mathbf{U}}_{t-1} + \boldsymbol{\zeta}_t; \\ \begin{pmatrix} \tilde{\mathbf{u}}_t \\ \tilde{\mathbf{u}}_{t-1} \\ \vdots \\ \tilde{\mathbf{u}}_{t-p+1} \end{pmatrix} &= \begin{pmatrix} \mathbf{z}_t \boldsymbol{\gamma}_t \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{A}_{1,t} & \dots & \mathbf{A}_{p-1,t} & \mathbf{A}_{p,t} \\ \mathbf{I} & & \mathbf{0} & \mathbf{0} \\ & \ddots & & \vdots \\ \mathbf{0} & & \mathbf{I} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{u}}_{t-1} \\ \tilde{\mathbf{u}}_{t-2} \\ \vdots \\ \tilde{\mathbf{u}}_{t-p} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\zeta}_t \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix}; \quad (6') \\ \tilde{\mathbf{U}}_t &= \boldsymbol{\mu}_t + \tilde{\mathbf{A}}_t \tilde{\mathbf{U}}_{t-1} + \tilde{\boldsymbol{\zeta}}_t. \end{aligned}$$

Using the multivariate lognormal density, we obtain for  $t = 1, \dots, p$

$$\begin{aligned} p(\mathbf{u}_1, \dots, \mathbf{u}_p | \boldsymbol{\mu}_{\tilde{U}_1}, \boldsymbol{\Sigma}_{\tilde{U}_1}) &= \\ &= (2\pi)^{-\frac{np}{2}} |\boldsymbol{\Sigma}_{\tilde{u}_1}|^{-\frac{np}{2}} \prod_{t=1}^p \prod_{j=1}^n \frac{1}{u_{tj}} \times \\ &\times \exp\left(-\frac{1}{2} (\ln \mathbf{U}_1 - \boldsymbol{\mu}_{\tilde{U}_1})' \boldsymbol{\Sigma}_{\tilde{U}_1}^{-1} (\ln \mathbf{U}_1 - \boldsymbol{\mu}_{\tilde{U}_1})\right), \end{aligned} \quad (8)$$

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<sup>9</sup>The original model can be recovered using the  $(n \times np)$  matrix  $\mathbf{J} = (\mathbf{I} \ \mathbf{0} \ \dots \ \mathbf{0})$ .

with

$$\begin{aligned}\boldsymbol{\mu}_{U_1} &= (\mathbf{I} - \tilde{\mathbf{A}}_1)^{-1} \boldsymbol{\mu}_1; \\ \text{vec} \boldsymbol{\Sigma}_{\tilde{U}_1} &= (\mathbf{I} - \tilde{\mathbf{A}}_1 \otimes \tilde{\mathbf{A}}_1)^{-1} \text{vec} \boldsymbol{\Sigma}_{\tilde{\zeta}}.\end{aligned}$$

For  $t = p + 1, \dots, T$ , we have

$$\begin{aligned}p(\mathbf{u}_t | \mathbf{u}_{t-1}, \dots, \mathbf{u}_{t-p}, \boldsymbol{\mu}_{\tilde{u}_t}, \boldsymbol{\Sigma}_{\tilde{u}_t}) &= \\ = (2\pi)^{-\frac{n}{2}} |\boldsymbol{\Sigma}_{\tilde{u}_t}|^{-\frac{n}{2}} \prod_{j=1}^n \frac{1}{u_{tj}} \exp \left( -\frac{1}{2} (\ln \mathbf{u}_t - \boldsymbol{\mu}_{\tilde{u}_t})' \boldsymbol{\Sigma}_{\tilde{u}_t}^{-1} (\ln \mathbf{u}_t - \boldsymbol{\mu}_{\tilde{u}_t}) \right),\end{aligned}\quad (9)$$

with

$$\begin{aligned}\boldsymbol{\mu}_{\tilde{u}_t} &= \mathbf{z}_t \boldsymbol{\gamma}_t + \mathbf{A}_t \ln(\mathbf{U}_{t-1}); \\ \boldsymbol{\Sigma}_{\tilde{u}_t} &= \boldsymbol{\Sigma}_{\tilde{\zeta}}.\end{aligned}$$

Combining the densities in (8) and (9) with the other priors and the likelihood in (3) leads to a density which does not have a recognizable functional form. Therefore, the sequence  $\mathbf{u}_t$  is generated in a Metropolis-Hastings step (Chib and Greenberg 1995).

## 4 Results

In this section, we present results for two regional aggregates, Centre-North and Mezzogiorno, in the observation period 1970-1994. To parameterize as parsimoniously as possible, we chose as a starting point a VAR of order 1. Therefore, equation (6) becomes

$$\begin{aligned}\tilde{\mathbf{u}}_t &= \mathbf{z}_t \boldsymbol{\gamma}_t + \mathbf{A}_t \tilde{\mathbf{u}}_{t-1} + \boldsymbol{\zeta}_t = \\ &= \mathbf{Z}_t \tilde{\boldsymbol{\alpha}}_t + \boldsymbol{\zeta}_t,\end{aligned}\quad (6'')$$

following the definitions from above. We draw a sample of 60'000 replications, and discard the first 10'000 as burn-in. The priors are given as  $\boldsymbol{\beta}_0 = \mathbf{0}$ ,  $\mathbf{B}_0 = 50 \times \mathbf{I}_k$ ,  $\nu_B = k + 1$ ,  $\boldsymbol{\alpha}_0 = \mathbf{0}$ ,  $\mathbf{P}_{\boldsymbol{\alpha}} = \mathbf{I}_m \times 1000$ ,  $\mathbf{V} = \mathbf{I}_m \times 10$ ,  $\nu_V = m + 1$ ,  $\boldsymbol{\Omega}_{\nu,0} =$

$\mathbf{I}_n \times 100, \nu_v = n + 1, \mathbf{\Omega}_{\zeta,0} = \mathbf{I}_n \times 1000, \nu_{\zeta} = n + 1, n = 2, k = 6, m = 10$ . The results for the posterior distributions of the production function parameters are displayed in Tables 1 and 2.<sup>10</sup>

Table 1: Production Function Parameters

	NSE using Autocovariance Taper				
	Mean	SD	4 per cent	8 per cent	15 per cent
$c_{CEN}$	-0.224	2.500	0.010	0.010	0.011
$c_{MEZ}$	-0.064	2.266	0.011	0.010	0.007
Trend	-0.092	0.540	0.003	0.003	0.002
Labour	0.645	0.849	0.005	0.005	0.004
Capital	0.721	0.735	0.004	0.004	0.004

The numerical standard errors (NSE) are calculated using the `coda.m` function in the econometrics toolbox fro Matlab provided by James P. LeSage ([www.spatial-econometrics.com](http://www.spatial-econometrics.com)).

Table 2: Error Variance-Covariance Matrix, Posterior Distribution

	NSE using Autocovariance Taper				
	Mean	SD	4 per cent	8 per cent	15 per cent
$\sigma_{CEN}$	0.016776	0.004228	0.000063	0.000062	0.000061
$\sigma_{MEZ}$	0.016931	0.004131	0.000096	0.000096	0.000089
$\sigma_{CEN,MEZ}$	-0.000017	0.000084	0.000001	0.000001	0.000001

The numerical standard errors (NSE) are calculated using the `coda.m` function in the econometrics toolbox fro Matlab provided by James P. LeSage ([www.spatial-econometrics.com](http://www.spatial-econometrics.com)).

A comparison of the posterior distributions with the prior of the production elasticities shows that despite of the small sample size, the prior chosen for this exercise is less informative than the data (Figure 1). The results for the time-varying VAR in efficiency are summarized in Figures 2 and 3. The maximum absolute eigenvalue of the VAR parameter matrix is an indicator for the persistence of shocks, as illustrated in the second panel of Figure

<sup>10</sup>The acceptance rate of the Metropolis-Hastings step was about 20 per cent. For the VAR parameter matrices, we obtained about 30 per cent of stationary draws after burn-in.

2, where the half-life of a shock is displayed.<sup>11</sup> In median, the maximum absolute eigenvalue varies over time between 0.4 and 0.6, which implies a half-life of about one year.

The time-varying long-run reaction of efficiency with respect to infrastructure investment is displayed in Figure 3.<sup>12</sup> With respect to non-core infrastructure, there is no difference between Centre-North and Mezzogiorno: the reaction is positive, and the time pattern exhibits an inverted U-shape, with upper turning point in about 1980. Core infrastructure has a negative effect in the North. In the South, the effect becomes positive after 1985.

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<sup>11</sup>Based on the maximum absolute eigenvalue  $\lambda_t$  at time  $t$ , we calculate half-life of a unit shock as

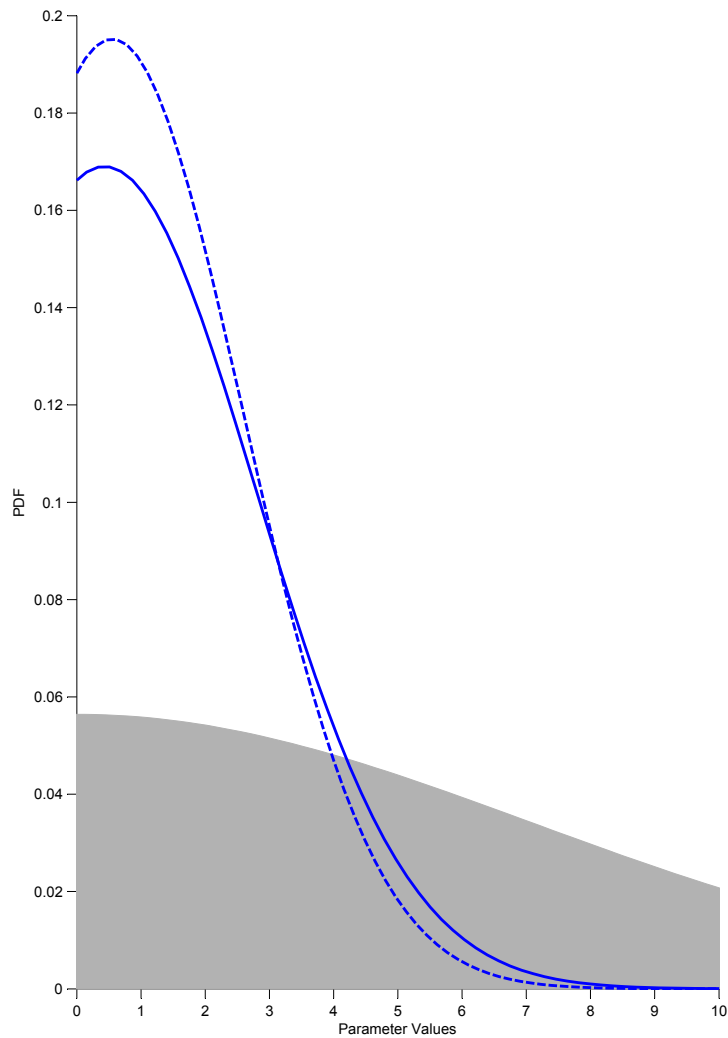
$$\text{half-life}_t = \frac{\log 0.5}{\log \lambda_t}.$$

<sup>12</sup>We calculate the long-run effect on  $\tilde{\mathbf{U}}_t$  as

$$\tilde{\mathbf{U}}_t = (\mathbf{I} - \mathbf{A}_t) \boldsymbol{\gamma}_t.$$

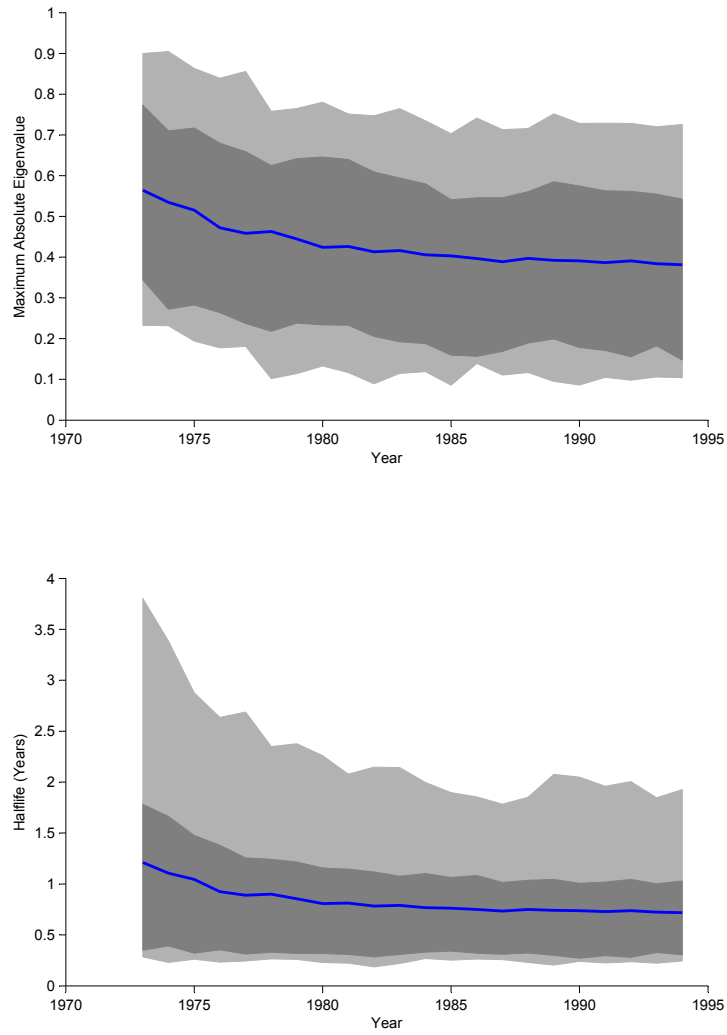
The effects displayed in Figure 3 are the reactions of efficiency, after the appropriate transformation of  $\tilde{\mathbf{U}}_t$ .

Figure 1: Distribution of Elasticities



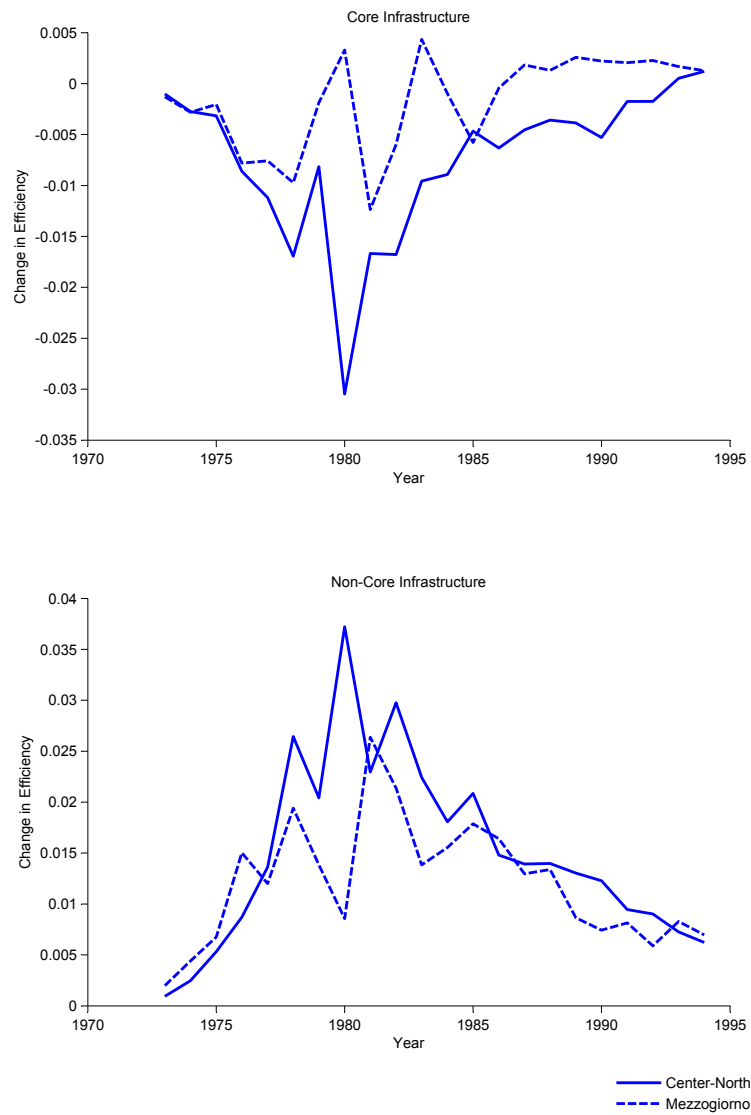
The shaded area indicates the prior distribution of the two elasticities, the solid curve is the production elasticity with respect to labour, and the dashed curve is the production elasticity with respect to capital. The posterior densities are estimated using a kernel density estimator (Gaussian kernel, see Pagan and Ullah 1999 for details).

Figure 2: Dynamics of the Efficiency VAR



The shaded areas are highest posterior density intervals for the maximum absolute eigenvalue of the parameter matrix and the implied half life of a shock (light grey: 90 per cent interval, dark grey: 68 per cent interval).

Figure 3: Median Long-Run Reaction of Efficiency to an Infrastructure Shock



## 5 Conclusion

The inclusion of short-term economic fluctuations is relevant to a stochastic frontier approach as it arises in an intertemporal context. The partial adjustment process continues through a number of periods, and is not en-

capsulated by the estimation of a frontier model based on the implicit assumption of complete adjustment. The failure to incorporate such partial adjustment into the model can lead to the inappropriate classification of an intertemporally efficient producer as being inefficient during the adjustment period.

In this paper, we report preliminary results from work in progress on stochastic frontier models with time-varying vector autoregressive inefficiency. We combine the approach by Battese and Coelli (1995) and Tsionas (2006) with recent work by Cogley and Sargent (2005) and Primiceri (2005): log-inefficiency follows a VAR with time dependent parameters. We apply this method to extend the analysis of different long-term growth paths in the Italian regions in Mastromarco and Woitek (2006), discriminating between the channels through which public infrastructure influences overall productivity. Our results are consistent with our previous empirical findings (Mastromarco and Woitek 2006). The empirical evidence shows that the long-run reaction of efficiency to core-infrastructure is bigger for the Southern regions than for North-Centre regions. Non-core infrastructure seems to have a more important long-run effect for Northern regions than Southern regions. In addition, we find that there is change over time in the strength of the relationship between public capital and efficiency. From 1975 to 1985 the reaction of efficiency to core infrastructure is the highest in the Southern regions; whereas the Northern regions show a downswing effect during 1975 to 1985 and upswing effect the rest of the period. The effect of non-core infrastructure to efficiency is increasing during 1970s and decreasing during 1980s and 1990s for both Southern and Northern regions.

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