

Long-Run Productivity Risk: A New Hope for Production-Based Asset Pricing?*

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Under Revision

Abstract

This study examines the intertemporal distribution of productivity risk. Using US data, I show that the conditional mean of productivity growth is both time-varying and extremely persistent. This generates uncertainty about the long-run perspectives of economic growth and affects significantly macro quantities and asset prices. After establishing this empirical link, I develop a production-based asset pricing model featuring long-run productivity growth uncertainty, adjustment costs, and recursive preferences à la Epstein-Zin. This model reproduces several features of both asset prices and macroeconomic quantities.

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1 Introduction

Aggregate productivity is one of the most important indicators of both the macroeconomic and financial conditions of a country. Fama(1981 and 1990), Cochrane (1996), and Balvers and Huang (2007), among others, have already documented the existence of a relevant link between the movement in asset prices and real economic activity at business cycle frequency. All of these studies, however, share one common feature: they do not distinguish the specific impacts that different sources of productivity uncertainty can have on stock prices. In particular, they do not disentangle the role of predictable fluctuations that are active at low frequency and explain productivity growth rate swings over long horizons. I refer to this type of fluctuation as long-run productivity risk, according to a similar terminology that has been adopted by Barsky and DeLong (1993), and more recently by Bansal and Yaron(2004).¹

In this paper, I consider the possibility that long-run consumption and dividend risks originate from a more primitive common variable: long-run productivity risk. In order to show the relevance of long-run productivity risk I do two things. First of all, I proceed with a new empirical investigation that documents the existence of a predictable component in productivity growth which affects both aggregate stock market prices and major macroeconomic variables like consumption, investment, output and dividends growth. Second, I propose a novel production based dynamic stochastic general equilibrium (DSGE) model featuring long-run productivity risk. This allows me to document the theoretical relevance that long-run productivity risk can effectively have.

To find evidence of the presence of long-run productivity uncertainty, I study the empirical intertemporal distribution of productivity risk for the US economy. I show that the conditional mean of annual productivity growth is time-varying and extremely persistent. This result is consistent with that found by Beaudry and Portier (2006), and Edge, Laubach and Williams (2007), who use real-time data to document time variation in the US long-run productivity growth forecast.² The slow fluctuations in the conditional mean of productivity growth produce uncertainty about the long-run perspectives of economic growth and affect asset prices. In particular, I show that stock market prices are very sensitive to long-run news for productivity growth: a 1% increase in the conditional mean of annual productivity growth is accompanied, on average, by an increase in the aggregate price-dividend ratio ranging between 11% and 50%.

The long-run component in productivity, however, also has a strong economic significance. Consumption, investment, labor, output and dividends all show a statistically significant positive exposure to long-run uncertainty. The growth rates of these variables are all predictable through

¹Barsky and DeLong (1993) show that a predictable and persistent component in dividends growth (namely, long-run dividend risk) can explain a large fraction of stock price fluctuations. Bansal and Yaron (2004) explore the role of long-run risk in consumption and dividends in a fully specified endowment economy with competitive markets. These authors, however, do not address the origins of the long-run component in consumption and in the assets' cash flow.

²Edge at al. (2007) also study the macroeconomic implications of this uncertainty factor in a production economy in which the representative agent has standard CRRA preferences and limited information about the long-run component.

the common long-run component in productivity. Together, short-run and long-run risk explain about 70% of the volatility of the macroeconomic variables mentioned above. Furthermore, I show that labor's sensitivity to long-run shocks is one order of magnitude greater than that to short-run news. This is yet another reason to believe that the intertemporal composition of productivity risk matters, and it has to be taken into account in order to better explain the joint behavior of prices and quantities.

After demonstrating an empirical link between productivity, asset prices and macroeconomic variables, I develop a production-based asset pricing model with complete markets featuring long-run uncertainty about the productivity growth rate. The main goal of this second step of my study is to explore in detail the theoretical implications of productivity growth predictability (long-run productivity risk) in the context of a fully specified model with production and recursive utility.

This theoretical approach is interesting because it bridges part of the gap between the current long-run risk literature and the macroeconomic literature. Standard production-based macroeconomic models try to explain the joint dynamics of quantities and productivity, focusing only on business cycle frequencies. In this literature, the implications for asset prices are usually neglected, and often they are simply counterfactual. In the finance literature, in contrast, long-run risk models are designed to explain key time-series properties of asset prices, keeping quantities as a given and focusing on the low-frequency fluctuations of cash flows and consumption. In this paper I propose a unifying framework in order to study the co-movements of asset-prices and quantities simultaneously over both the short and the long horizon.

For the sake of simplicity, the model is extremely parsimonious. It considers an infinitely lived representative agent with Epstein-Zin (1989) and Weil (1989) preferences. These preferences are important because they disentangle the intertemporal elasticity of substitution (IES) from the relative risk aversion coefficient (RRA) and are sensitive to the intertemporal distribution of risk.

The representative agent supplies labor and accumulates capital subject to adjustment costs. As a consequence, in this model the supply curve of new capital is not perfectly flexible; the price of new capital is time-varying and increases when investment rises. To my knowledge, I propose a new adjustment cost formulation which differs from that one adopted by Jermann (1998). In my model, the cost depends on the adjustment in the composition of output. In particular, the adjustment cost is increasing in the gap between the actual value and the unconditional mean of the investment-output ratio. The relevance of this cost specification is twofold: it allows to have sizable fluctuations in both prices and quantities, and it seems empirically plausible. This adjustment cost specification, indeed, predicts a positive relationship between the value of capital and the investment-output ratio. Focusing on US annual data, I show that such positive relationship is statistically significant.

Productivity growth is exogenous and is affected by two different sources of uncertainty: a short-run shock that is *i.i.d.* (standard in the real business cycle literature), and a long-run

component that is responsible for small but persistent fluctuations in the drift of productivity. The latter component is calibrated to introduce long-run productivity swings.

Under this set-up, the model successfully reproduces key features of both asset prices and macroeconomic quantities such as consumption, investment, and output. This is significant because the reconciliation of asset market factors with aggregate quantities behavior has proved a challenge for modern DSGE models. Rouwenhorst (1995), for example, shows that in a model with production and standard time-additive CRRA preferences as in Kydland and Prescott (1982), it is impossible to reproduce the US historical equity premium. Jermann (1998) shows that introducing adjustment costs in a model with CRRA preferences is not enough to produce relevant premia. At the same time, Jermann shows that when habit preferences and adjustment costs are combined, it is possible to obtain high equity premia and a low risk-free rate in an economy with production and fixed labor supply. Although this result represents a very important improvement in the production-based asset pricing literature, it is not completely satisfactory. As pointed out by Boldrin, Christiano and Fisher (2001) and Lettau and Uhlig (2000), Jermann's model produces a countercyclical response of labor to a persistent shock to productivity which is at odds with the data. Boldrin, Christiano and Fisher (2001) solve this problem by proposing a two-sector economy with limited labor and investment mobility. Contemporaneously, however, their model predicts a negative serial correlation for consumption growth and an excessively volatile risk-free rate. This is indeed a common problem shared by most models with habit preferences.

Given all these documented difficulties, it is remarkable that the model presented in this paper is able to reproduce several key facts about both the equity premium and the risk-free rate. In particular, the model produces a sizable equity premium and a low average risk-free rate, with a moderate amount of RRA and an IES slightly larger than one. While in common production economies the premium is generated through an unrealistic high contemporaneous correlation between returns and consumption growth, in my production economy this correlation is very low, as in the data. Furthermore, the model delivers an implied risk-free rate as persistent as is observed in the data, while the excess returns are instead almost unpredictable, again consistent with the empirical evidence.

Although these results look very promising, it has to be acknowledged that the returns volatility puzzle remains unexplained in my framework. Embodied in the model there is a relevant trade-off between the volatility of the quantities and that of asset returns. Such a trade-off is generated by the need of a real friction that allows for time-variation in the marginal price of capital. The more severe the friction is, the less volatile quantities are. On the other hand, the weaker the friction, the smoother the price-dividend ratio. In order to match the volatility of the quantities, I calibrate the adjustment costs to be very small.³ Thanks to the introduction of the long-run component, however, I am able to produce non-negligible fluctuations in the stock price even with a very mild friction. The long-run shocks, in fact, are able to produce

³At the equilibrium, the implied average adjustment cost is about .06% of output, a very small number.

substantial shifts in the demand of new capital and generate relevant price movements even if the supply curve is not very steep or, in other words, even if the adjustment costs are not strong. As a matter of fact, in my benchmark calibration the average adjustment costs are about 10 times smaller than those in Jermann (1998).

The performance of the model about second moments is definitely more convincing when we focus on contemporaneous correlations. The model is able to produce the right amount of co-movements not only between returns and real variables, but also across the macro variables. A well known problem with standard real business cycles model is that they produce an almost perfect correlation between consumption and investment. In the data, however, the contemporaneous correlation between consumption and investment growth is about 45% at quarterly frequencies and about 70% at an annual frequency. Under the benchmark calibration, I show that when the IES is high enough, the substitution effect dominates the income effect: good news for the expected productivity growth rate provides a strong incentive to reduce consumption and invest more. Meanwhile, good news for the short-run implies an increase in both consumption and investment. The overall correlation between consumption and investment is positive and clearly in line with the data.

To my knowledge, this paper is one of the first to study the interaction between predictability in productivity growth and recursive preferences. For this reason I devote the final section of the paper to the influence of the IES and the adjustment costs on both quantities and prices. I show that the IES has an extremely important impact on the dynamics of both quantities and prices mostly in response to long-run shocks.⁴ I also explore the role of adjustment costs by comparing my benchmark model to one in which there is no adjustment cost, and one in which I adopt Jermann’s adjustment costs. This experiment is particularly interesting because it relates asset-price implications to very different consumption-smoothing possibilities. I point out two relevant results: (i) a real friction on investment is absolutely required in order to let the Bansal and Yaron results hold also in a production economy; and (ii) my adjustment costs produce better results than those obtained by adopting Jermann’s adjustment costs formulation.

1.1 Related literature

This paper is intended as a contribution to the long-run risk literature, the real business cycle (RBC) literature, and the literature regarding “production-based asset pricing” (Cochrane, 2005).⁵ The most important papers related to this work are those of Tallarini (2000), Bansal and Yaron (2004) and Beaudry and Portier (2006).

Tallarini (2000) is the first to employ risk-sensitive preferences in a production economy model in order to study the joint dynamics of prices and quantities. He works with a repre-

⁴The RRA, in contrast, has only a marginal impact on quantities, while it has a strong impact on both the risk-free rate —because of precautionary savings motives— and the equity premium. See Tallarini (2000).

⁵For the sake of brevity, in this section I discuss only a subset of papers that are closely related to mine. This section is not – and does not aim to be – a complete review of the aforementioned literatures.

sentative agent that has Epstein-Zin-Weil preferences, but he focuses only on the case in which the intertemporal elasticity of substitution is one.⁶ Furthermore, productivity in his model follows a random walk with a constant drift, and the marginal price of new capital is fixed at one. He shows that his model has implications for macroeconomic quantities comparable to those obtained by Kydland and Prescott (1982) and that, for a given IES, the RRA has only second-order effects on this result. At the same time, Tallarini’s model does not solve the equity premium puzzle, meaning that extremely high values of RRA (even 100) are required in order to justify the high Sharpe Ratio observed in the data. I show that this class of models can produce significantly improved results once we recognize the relevance of the following three additional economic factors: (1) the existence of predictability in productivity, (2) the relevance of time variation in the marginal value of capital, and (3) the interaction between the IES and income effect generated by future productivity shocks.

Bansal and Yaron (2004) demonstrate that in an exchange economy with long-run risk in consumption and dividends cash flow and Epstein-Zin-Weil preferences, it is possible to reconcile consumption and asset-price properties with moderate risk aversion and an IES slightly greater than one. I incorporate their insight in a simple production economy with adjustment costs on the composition of output in which the long-run risk is a primitive component of productivity. In my model, the long-run productivity risk optimally generates endogenous long-run risk in consumption and dividends and, as a consequence, high equity premia. More generally, analyzing this production economy allows me to bridge part of the gap between the current long-run risk literature, which takes consumption and dividends as given, and the business cycle literature, which often neglects the role of asset prices.

Beaudry and Portier (2006) have already documented the existence of an empirical link between asset prices and productivity. I confirm their finding and I provide further empirical evidence about the role of the long-run component. My empirical analysis is novel in three respects. First, it includes also aggregate dividends and shows that they are 3 times more exposed to long-run risk than consumption. Second, it takes simultaneously into account variations in both conditional mean and volatility of productivity. Third, it adopts a different econometric specification that better allows to test the implications of long-run risk models. Additionally, I use a fully specified structural DSGE model in order to explicitly study the role of the deeper parameters of the model.

The interaction between long-run risk and production-based DSGE models has recently attracted significant attention.⁷ In an independent and contemporaneous paper, Campanale, Castro and Clementi (2006) characterize asset returns in a production economy with capital adjustment costs, trend-stationary productivity, and generalized recursive preferences à la Epstein-Zin

⁶This allows Tallarini to solve his model simply by applying the discounted linear exponential quadratic Gaussian control methodology developed by Hansen and Sargent (1995).

⁷My analysis of the long-run productivity risk in the context of a production economy started in 2005. The current manuscript is based on my job market paper “Welfare Costs and Long-Run Consumption Risk in a Production Economy.”, presented for the first time at the E.C.B. Macro-Seminars, August 2005.

(1989) and Gul (1991).⁸

In another independent and contemporaneous paper, Kaltenbrunner and Lochstoer (2006) work with a production economy with convex capital adjustment costs, and generalized recursive preferences à la Epstein-Zin. They show that in this environment, capital accumulation generates endogenous long-run consumption risk—a result confirmed in my investigation—and that the consumption-productivity ratio is an important factor of risk. They work with just one source of productivity uncertainty and do not estimate long-run productivity risk. In this paper, however, I show that by neglecting the long-run shock we could miss two important elements. First of all, the long-run shocks give us crucial information on the role of the IES in determining the co-movements of investment, consumption, and returns. Second, by neglecting the long-run uncertainty channel we could substantially underestimate the true market price of risk.

In a recent working paper, Backus, Routledge and Zin (2007) model a production economy with Epstein-Zin preferences and generalized predictability in productivity growth in order to reproduce the intertemporal co-movements of bond prices and quantities over the business cycle.⁹

Ai (2007) studies the asset-pricing implications of long-run productivity risk in a production economy in which the representative agent has limited information about the long-run component and has access to an AK technology function.¹⁰ In such an economy, dividends equal total output. In my model, in contrast, total output and dividends differ from each other because of labor income and financial leverage.¹¹

Finally, Uhlig (2007) explores the connection between leisure and the market price of risk in a DSGE model with recursive preferences.

2 Empirical evidence

I denote the level of productivity at time t as A_t . I use lowercase letters for log-units. In order to study the intertemporal distribution of risk related to productivity, I focus on the following model:

$$\begin{aligned} \Delta a_{t+1} &= \mu + x_t + \sigma \epsilon_{a,t+1} \\ x_t &= \rho x_{t-1} + \sigma_x \epsilon_{x,t} \\ \begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} &\sim iidN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right). \end{aligned} \tag{1}$$

⁸When working with Epstein-Zin preferences, Campanale, Castro and Clementi (2006) calibrate their IES to a number very close to zero in order to match the relative volatility of investment and consumption. At the same time, however, their model produces a risk-free rate that is excessively volatile.

⁹A previous version of their paper was circulated under the title “Leads, Lags, and Logs: Asset Prices in Business Cycle Analysis.”

¹⁰An earlier version of Ai’s paper was circulated under the title “Incomplete Information and Equity Premium in Production Economies” (2005).

¹¹In this version of the model I keep the labor supply constant. The basic intuitions behind the model do not change when I allow labor to be endogenously time-varying. These results are available upon request.

The long-run risk component (LRR), x_t , captures persistent fluctuations in the conditional mean of the productivity growth rate. The short-run risk (SRR), $\epsilon_{a,t+1}$, instead is *i.i.d.*

In the following subsection I estimate the long-run component using Kalman-filter. In section 2.2, instead, I use price-dividends news in order to disentangle the two shocks $\epsilon_{a,t+1}$ and $\epsilon_{x,t+1}$.

2.1 Univariate Approach

Hamilton (1994) shows that to estimate the state-space (1) by Kalman-filter with constant Kalman-gains is equivalent to estimate the following ARMA(1,1):

$$\begin{aligned}\Delta a_{t+1} &= \mu(1 - \rho) + \rho\Delta a_t - b\sigma_v v_{a,t} + \sigma_v v_{a,t+1} \\ v_{a,t+1} &\sim iidN(0, 1).\end{aligned}\tag{2}$$

The productivity growth rate can be rewritten in the following way:

$$\begin{aligned}\Delta a_{t+1} &= \underbrace{x_{v,t}}_{LRR} + \underbrace{\sigma_v v_{a,t+1}}_{SRR} \\ x_{v,t} &= E_t[\Delta a_{t+1}] = \mu(1 - \rho) + \rho\Delta a_t - b\sigma_v v_{a,t}.\end{aligned}\tag{3}$$

$$\tag{4}$$

The filtered long-run risk component, $x_{v,t}$, captures persistent fluctuations in the conditional mean of the productivity growth rate. The short-run risk, $v_{a,t+1}$, instead is *i.i.d.*. Notice that the Wold innovations $v_{a,t+1}$ are a combination of the entire history of the two shocks $\epsilon_{a,t+1}$ and $\epsilon_{x,t+1}$. Since I am only using data on productivity I am not able to identify these two different shocks. The current approach, however, provides me a first chance to test whether there is a predictable component in productivity growth.

I estimate equation (2) using post-war annual data. I use a multifactor productivity index that takes into account capital accumulation and has already been studied by Beadry and Portier (2006).¹² Working with annual data is particularly important for two reasons. First, the data are not altered by any seasonal adjustment; second, they are more likely to contain a better signal and less noise related to the low-frequency component of productivity. This kind of approach is not new to the long-run risk literature. Harvey (1986) and, more recently, Bansal, Kiku and Yaron (2006) and Colacito and Croce (2007) use annual data to estimate long-run risk in consumption. I follow the same methodology, applying it to productivity.

In Table 1 I report my estimates for productivity growth. In the top panel I estimate the ARMA(1,1) model, assuming that the innovations are homoskedastic. This simple model explains about 25% of the total variance of productivity growth. Furthermore, the likelihood-ratio test rejects the null hypothesis that productivity growth is perfectly *i.i.d.* The forecast

¹²Data are from the Bureau of Labor Statistics; the sample spans 1948–2003. In particular, the index measures the value-added output per combined unit of labor and capital input in private business and private nonfarm business. Available at <ftp://ftp.bls.gov/pub/special.requests/opt/mp/prod3.mfptablehis.zip>

for productivity is extremely persistent: using the adjusted sample for the years 1951–2003, the implied annual persistence is about .84.

Several recent papers have documented a reduction in the volatility of aggregate variables.¹³ To control for predictable time-variation in the volatility of productivity growth, I estimate the following model that includes a GARCH(1,1):

$$\begin{aligned}\Delta a_{t+1} &= \mu(1 - \rho) + \rho\Delta a_t - b\sigma_{v,t}v_{a,t} + \sigma_{v,t+1}v_{a,t+1} \\ \sigma_{v,t+1}^2 &= \kappa + \rho\sigma_{v,t}^2 + b\sigma_{v,t}^2v_{a,t}^2\end{aligned}\tag{5}$$

In the lower portion of Table 1 I report the implied estimates. The predictable component continues to be extremely persistent, although less volatile, and it explains about 10% of the total variance of productivity growth.¹⁴ The likelihood-ratio test confirms that we cannot reject the existence of predictability at a confidence level of 10%. In Fig. 1 I plot the productivity growth rates and the forecasts obtained from the ARMA(1,1) model with and without the GARCH(1,1) correction. The models give similar results: the conditional mean of productivity slowly declines until the 1970s, and then it begins to grow.

Both coefficients of the GARCH(1,1) are statistically different from zero. In Fig. 2 I depict the implied conditional volatility of productivity growth. The conditional volatility spikes up during the 1974 and 1983 crises, and then slowly declines. The flat, thin line represents the unconditional volatility of productivity growth measured on the whole sample. Since 1990, the conditional volatility of productivity has been moderate and has remained below its historical average. These findings are consistent with that observed for other macroeconomic quantities.

I now turn my attention to the impact of news about the conditional mean of the productivity growth rate on the aggregate stock market price. I use monthly CRPS value-weighted returns in order to construct my price-dividend ratio. Dividends are time-aggregated to an annual frequency, prices are measured at the end of the year. Both the aggregate stock market price and the long-run component in productivity are very persistent. They evolve as AR(1). In order to measure the influence of news to these two variables, I use the residuals of the following OLS regressions:

$$\begin{aligned}p_t - d_t &= \beta_{0,pd} + \rho_{pd}(p_{t-1} - d_{t-1}) + \epsilon_{pd,t} \\ x_t &= \beta_{0,x} + \rho_x x_{t-1} + \epsilon_{x,t},\end{aligned}\tag{6}$$

¹³See, among others, Lettau, Ludvigson and Wachter (2006), Fernandez-Villaverde and Rubio-Ramirez (2007), and Juliano e Primiceri (2008).

¹⁴In the sample I consider, the estimate of the MA root, \hat{b} , is greater than one. This would imply that the MA component of productivity growth is not invertible. In the Technical Appendix, I show that when both the predictable component in productivity growth and the sample size are small, this kind of result is quite frequent, even if the true data generating process has $b < \rho < 1$.

where x_t is the ARMA(1,1) forecast of productivity growth. In a second step I estimate:

$$\epsilon_{pd,t} = \beta_0 + \beta_1 \epsilon_{x,t} + \epsilon_t. \quad (7)$$

The estimates are reported in Table 2. I find a statistically significant positive relationship between price-dividend news and long-run productivity news. When I estimate the long-run component without including GARCH effects, I find that a 1% positive shock to long-run productivity can raise the price-dividend ratio by 11%. When I control for GARCH effects, the price-dividend ratio becomes even more sensitive, and $\hat{\beta}_1 = 22$. This result is consistent with the idea of Barsky and DeLong (1993) and Bansal and Yaron (2004) that small but persistent innovations to the economy growth rate can produce a substantial adjustment in asset prices. This is an important result that motivates the next empirical analysis.

2.2 Bivariate Approach

The previous section shows that the price-dividends ratio contains information about the long-run productivity component. In order to be able to estimate both the structural parameters and the specific shocks in (1), I then introduce information contained in the log price-dividends time-series:¹⁵

$$\begin{aligned} \Delta a_{t+1} &= x_t + \sigma \epsilon_{a,t+1} \\ x_t &= \rho x_{t-1} + \sigma_x \epsilon_{x,t} \\ pd_{t+1} &= \rho_{pd} pd_t + \alpha_1 \sigma_x \epsilon_{x,t+1} + \alpha_2 \sigma \epsilon_{a,t+1} \\ \begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} &\sim iidN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right). \end{aligned} \quad (8)$$

After initializing the long-run component at its unconditional mean, for a given set of parameters $\{\rho, \rho_{pd}, \alpha_1, \alpha_2\}$ I can recursively recover the entire time-series of both the short-run and the long-run news. At this point, a simple GMM estimation—described in the Technical Appendix—allows me to jointly estimate the variances and the correlations of the two shocks, $\{\sigma, \sigma_x, \rho_{xa}\}$, together with all the other structural parameters.

I report the results from the GMM estimation in Table 3. The first thing to notice is that the identifying parameter σ_x is precisely estimated and statistically different from zero. This result confirms the existence of predictability in productivity growth and lets all the other parameters be identified. The long-run component persistence is very close to that of the price-dividend ratio. Furthermore, the price-dividend ratio is extremely sensitive to long-run shocks. As predicted by the long-run risk paradigm, good news for the long-run implies a strong positive innovation in the price-dividends ratio. On the other side, good news for the short-run has a small negative effect on the stock prices. In the next section I show that both these two different

¹⁵In this section all the results are produced using demeaned variables.

responses of the price-dividends ratio are going to be qualitatively replicated by the theoretical model.¹⁶

According to my identification scheme, long-run and short-run shocks have a contemporaneous correlation which is not statistically different from zero. The short-run shocks are small and have a standard deviation that is about 15% of that of the short run-shocks. The implied amount of variance of the productivity growth rate explained by the estimated long-run component is overall 8%.

In Fig. 3 I show the long-run component estimated according to (8), the forecast obtained by the ARMA(1,1), and the productivity growth rate. The two long-run components follow similar paths and their contemporaneous correlation is about 66%. In particular, both estimation procedures capture the fall of the productivity conditional mean during the 1970s, and its subsequent rise. This is a reinsuring result: using information from asset prices refines the estimation of the long-run component without altering its basic time-series properties.

Finally, in Fig. 4 I show the estimated conditional volatility for both the short-run and the long-run news to productivity. I obtain these time-series by estimating a GARCH(1,1) on the fitted shocks $\hat{\epsilon}_{a,t+1}$ and an ARCH(1) on $\hat{\epsilon}_{x,t+1}$. I report the estimation results below the graphs. In the right panel of the figure we can see that the conditional volatility of the short-run shocks has declined over time, as has that of the aggregate productivity growth rates. The estimates of the GARCH(1,1) parameters confirm that this decline is statistically significant. In the left panel of Fig. 4, instead, we can see that the long-run shocks show just a marginally significant variation in their conditional volatility. These two graphs are relevant because they suggest that the behavior of macroeconomic uncertainty has been very different across the short-run and the long-run component of productivity. While short-run uncertainty has gone down over time, long-run uncertainty has remained pretty unchanged.

2.3 Macroeconomic significance

What is the role of short-run and long-run risk for macroeconomic variables such as output, investment, consumption, labor and dividends? Is the long-run risk component relevant only for asset prices, or is there a connection with macroeconomic fundamentals? In order to answer these questions, in this section I explore the explanatory power of both short-run and long-run risk on the growth rates of the macroeconomic variables listed above. The main results are reported in table 4.

The first thing to notice is that the long-run component in productivity has a significantly positive impact on consumption, investment, output and dividends. Output is less exposed to both short-run and long-run shocks than investment, but it is more exposed than consumption.

¹⁶Although broadly compatible, the results of this sections are not directly comparable to those of Beaudry and Portier (2006). They estimate a VAR/VECM on TFP growth and the change in the S&P500 index. I use a price-to-fundamentals measure from CRSP (the P/D ratio) and a state space representation familiar to the long-run risk literature.

Furthermore, dividends have an exposure to long-run risk that is about three times bigger than that of consumption, exactly as in the Bansal and Yaron (2004) calibration.

In a basic production economy in which labor is perfectly flexible, the growth rate of labor would depend on the growth of the two exogenous components. For this reason, I first regress labor growth on $\epsilon_{a,t}-\epsilon_{a,t-1}$ and $x_{t-1}-x_{t-2}$ (regression 4). In this case I obtain a strong positive dependence of labor on long-run shocks, while the short-run shock differential seems to play no role. It is well known, however, that the labor market is affected by important real frictions that generate a delayed response to productivity news. In order to take this into account, I also project labor growth on $\epsilon_{a,t}$ and $\epsilon_{a,t-1}$ separately (regression (5)). In this case both the contemporaneous short-run shock and its first lag become strongly significant. Labor is procyclical with respect to short-run news and it adjusts slowly over time. Moreover, both regression (4) and (5) confirm that long-run shocks have an impact on labor growth one order of magnitude greater than that of short-run shocks.¹⁷ This is an interesting fact suggesting that to disentangle the long-run component of productivity growth is necessary to better understand the dynamics of quantities.

The existence of predictability in productivity growth and its implications for consumption, investment, and output represent a crucial empirical result that motivates and supports the model explored in the next sections.

3 The model

This section presents the model used to examine the link between productivity, asset prices, and other macroeconomic fundamentals. To keep the analysis as simple as possible, I focus only on the representative agent consumption-saving problem, and I keep constant the labor supply. The representative agent has preferences defined only by aggregate consumption:

$$U_t = \left[(1 - \delta)C_t^{1-\frac{1}{\Psi}} + \delta \left(E_t [U_{t+1}^{1-\gamma}] \right)^{\frac{1-\frac{1}{\Psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\Psi}}} .$$

$$0 \leq C_t$$

Output is produced according to a constant returns-to-scale neoclassical production function:

$$Y_t = K_t^\alpha [A_t n_t]^{1-\alpha} ,$$

where K_t is the fixed stock of capital carried into date t , n_t is the labor input at t , and A_t is an aggregate productivity shock. The productivity growth rate, $\Delta a_{t+1} \equiv \log(A_{t+1}/A_t)$, has a long-run risk component and evolves as in (1). Because the main goal of this study is to examine the role of uncertainty about the conditional mean of productivity growth, I assume that the

¹⁷If we consider the regression $\Delta n_t = \beta_0 + \beta_1 \epsilon_{a,t} + \beta_2 \epsilon_{a,t-1} + \beta_3 x_t + \beta_4 x_{t-1}$, according to the Likelihood-ratio test, it is not possible to reject that $\beta_3 = -\beta_4$.

volatility of the shocks to productivity is constant.¹⁸ I also assume that the representative agent has full information about the two components of productivity growth.¹⁹ The other constraints in this economy are:

$$\begin{aligned} Y_t &\geq C_t + I_t, & \bar{n} &\geq n_t + l_t, \\ K_{t+1} &\leq (1 - \delta_k)K_t + I_t - G\left(\frac{I_t}{Y_t}\right)K_t, \\ G\left(\frac{I_t}{Y_t}\right) &= \left[a_1 \left| \frac{I_t}{Y_t} - \frac{\bar{I}}{\bar{Y}} \right| \right]^\tau. \end{aligned} \tag{9}$$

The agent is endowed with \bar{n} units of time that she can devote to leisure (denoted by l_t) or labor. Since leisure does not appear in the utility function, the representative agent will always find it optimal to offer $n_t = \bar{n}$ units of labor.²⁰ The rate of depreciation of capital is denoted by δ_k , and the function $G(\cdot)$ takes into account adjustment costs in the composition of output. I discuss in detail these adjustment costs in section 4.3.

3.1 Equilibrium

In this economy, it is possible to find the competitive equilibrium allocation by solving the planner's problem. Once the planner's allocation is found, prices and returns can be derived as follows.²¹ The stochastic discount factor takes the following usual form:

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\Psi}} \left(\frac{U_{t+1}}{E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\Psi} - \gamma} \tag{10}$$

The second factor relates to news regarding the continuation value of the representative agent. Future utility is very sensitive to long-run news, and for this reason it can induce high volatility in the stochastic discount factor even for moderate amounts of risk aversion.²² The risk-free rate is $R_t^f = E_t[M_{t+1}]^{-1}$. The marginal value of capital, q_t , is equal to the marginal rate of

¹⁸Fernandez-Villaverde and Rubio-Ramirez (2006) study the macroeconomic implications of time-varying volatility in productivity growth in the context of a production-based model with standard preferences.

¹⁹There are several relevant papers that study the role of limited information about the long-run component. See, among others, Edge, Laubach and Williams (2007); Ai (2007); Croce, Lettau and Ludvigson (2005); Hansen and Sargent (2006); and Bansal and Shaliastovich (2008).

²⁰I impose $\bar{n} = .18$. Like Tallarini (2000), I consider total employment multiplied by average weekly hours worked divided by the civilian non institutional population 16 years and older.

²¹See Sargent and Ljungqvist (2004) for the decentralization. My computational methods are described in detail in the Technical Appendix.

²²For the interaction between Epstein-Zin preferences and long-run consumption risk, see, among others, Bansal and Yaron (2004); Hansen, Heaton and Li (2005); Croce (2006).

transformation between new capital and consumption:

$$q_t = \frac{1}{1 - G'(\frac{I_t}{Y_t})\frac{K_t}{Y_t}}.$$

The returns per unit of normalized capital are

$$R_{t+1} \equiv \frac{q_{t+1} + D_{t+1}}{q_t},$$

where

$$D_{t+1} \equiv \alpha \frac{Y_{t+1}}{K_{t+1}} - \delta_k q_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} G\left(\frac{I_{t+1}}{K_{t+1}}\right).$$

I document results on levered market returns by introducing constant financial leverage in accordance with Boldrin, Christiano and Fisher (1995).²³ Wages equate the marginal productivity of labor.

3.2 Benchmark calibration

As is typical in the long-run risk literature, I assume the representative agent has a monthly decision horizon. In accordance with Bansal and Yaron (2004) methodology, I calibrate the model to a monthly frequency and then I focus on time aggregated annual statistics.

Productivity growth in the US has an annual average of 2% and an annual volatility of 2.9%. For this reason, I set $\mu = .00165$, and $\sigma = .008$. The long-run component in productivity is calibrated so as to be relatively small but persistent, as seen in the previous section. I impose $\rho = .98$, a conservative and empirically plausible number that in annualized terms implies a persistence of .80. To keep the long-run component as small as in the data I need to impose $\sigma_x = 5.5\%\sigma$.

The annualized capital depreciation rate is 6%. The parameter α is calibrated to match the capital income share. The parameter τ is calibrated to have a relatively sensitive Tobin's q ; a_1 is calibrated to make the adjustment costs very small on average (.06% of output), so that investment can be volatile enough.

In order to match the historical equity premium, the RRA is set to a value of 30. This is a plausible value, given the recent empirical findings in Bansal, Kiku, and Yaron (2006); Attanasio and Vissing-Jorgensen (2003); and Malloy, Moskowitz, and Vissing-Jorgensen (2006), but it is still reasonably low in light of the well-known difficulties of producing high equity premia in production economies.²⁴ Tallarini (2000), for example, using a risk aversion value

²³The levered excess return is: $R_{t+1}^d - R_t^f \equiv (R_{t+1} - R_t^f) \left(1 + \frac{\bar{B}}{\bar{S}}\right)$, where $\frac{\bar{B}}{\bar{S}}$ is the average debt-share ratio of the firm, and is estimated to be about 2/3.

²⁴Malloy, Moskowitz and Vissing-Jorgensen (2006) exploit micro-level household consumption data and show that aggregation across stockholders affects the estimate of the RRA coefficient. When the authors focus only on wealthy stockholders, the RRA estimate is 10. When they consider all the stockholders, the RRA is about 20. When they use aggregate consumption, this estimate increases even more.

of 100, obtains an annualized equity premium of just .04%, about 100 times smaller than that observed in the data. A similarly moderate level of relative risk aversion is adopted also by Lettau, Ludvigson and Wachter (2006). In my benchmark calibration, I fix the IES at 2. This value is consistent with that estimated by Basal, Gallant, and Tauchen (2007); Colacito and Croce (2007); Bansal, Kiku, and Yaron (2006); and Attanasio and Vissing-Jorgensen (2003). Finally, the annualized subjective discount factor is fixed at .98.

3.3 Predictions on quantities and prices

The main results produced by this calibration are reported in Table 5. In panel B I report first and second moments for both quantities and prices, while in panel C I focus on their co-movements.

The model reproduces quite well the observed unconditional mean of the growth rate of both investment and consumption. Furthermore, it implies an average income-output ratio of 29%, a number very close to that observed in the data. Models with standard time-additive CRRA preferences are not able to match the low level of the investment-output ratio and the low level of the risk-free rate simultaneously. Thanks to the Epstein-Zin preferences, however, this is not a problem. Under the benchmark calibration, the unconditional average of the risk-free rate is 1.1%, consistently with the data.

The implied equity premium, in log-units, is 3%. Although this value is half of that the historical mean of the CRSP stock market excess returns, it should be considered as a relevant accomplishment relative to the literature for at least four reasons. First of all, I am working with post—World War II data and I am keeping the volatility of consumption growth to 1.5%, half of that targeted by Bansal and Yaron (2004) and Kaltenbrunner and Lochstoer (2006). Second, this is a remarkably high number compared to that obtained by Tallarini (2000). Third, Bansal and Yaron (2004) in their exchange economy show that one third of the observed premium could be generated by uncertainty in volatility, a channel that is not active in my model. Finally, 3% is just slightly below the estimates reported by Fama and French (2001).

To understand the reasons that the model is able to generate such a high equity premium, note that the Euler equation implies that $E_t[r_{t+1} - r_{f,t}] \approx -cov_t(m_{t+1}, r_{t+1})$. Capital must offer a higher equity premium when its returns move in a direction opposite to that of the discount factor. In Fig. 5 I show the impulse response function of both quantities and asset prices after short- and long-run shocks in order to better highlight their co-movements.

The plots show the percentage deviations from the steady state of the monthly growth rates realized after a single positive pulse shock to both the short-run component (left panels) and the long-run risk (right panels). The shocks materialize only at time 2, and they are normalized according to their standard deviations ($\epsilon_{a,2} = \sigma$ and $\epsilon_{x,2} = \sigma_x$).

When a short-run shock materializes, the representative agent finds it optimal to increase consumption and investment at the same time. Investing more allows the representative agent to temporarily increase the capital stock and smooth consumption over time. This impulse

response is fully consistent with the predictions of any standard RBC model.

The behavior of consumption and investment is very different with respect to the long-run component. First of all, notice that long-run shocks are very long-lasting, and for this reason they can have a very strong impact on savings decisions even if they are small. Furthermore, long-run news simultaneously generates both an income effect and a substitution effect, which work in opposite directions. Higher expected long-run productivity generates a substitution effect that increases the opportunity cost of consumption, which tends to stimulate investment. Since output is predetermined, an increase in investment generates a contemporaneous drop in consumption. At the same time, a positive long-run shock allows the agent to feel much richer and to desire an immediate increase in consumption. The long-run component, in fact, is highly persistent, and thus a single long-run shock is able to affect the flow of expected future utility over a very long time-horizon. Indeed, a positive long-run shock translates into a remarkable increase in the continuation value of the agent. For a given output, the income effect tends to produce an increase in consumption and a drop in investment.

When the IES is high enough, as in the benchmark calibration, the degree of substitutability between continuation value and current consumption is also high. In this case, the substitution effect dominates the income effect, and the agent finds it optimal to decrease consumption in order to accumulate more capital. This is the reason that after a positive long-run shock, consumption growth drops while investment growth increases (Fig. 5, right panels).

Under the benchmark calibration, investment rises both after positive long-run and short-run shocks. This produces a pressure on the price of capital to appreciate, so that both short-run and long-run shocks imply bigger stock market returns and a contemporaneous fall in the stochastic discount factor. The stock market is indeed risky with respect to both sources of uncertainty, and for this reason it pays the high equity premium reported in Table 5.

The third column in panel B focuses on the volatility of both quantities and prices. The volatility of the annual consumption growth is 1.5%, consistent with the post-war data. Investment volatility is 5.1%, again an empirical plausible value.

The excess returns implied by the model are unfortunately much less volatile than those observed in the data. This is due to the fact that both the price-dividend ratio and dividend growth are not volatile enough. Overall, however, thanks to the presence of adjustment costs, the volatility is much higher than that observed in standard frictionless production-based models.²⁵ It is important to underline two things in favor of the model. In Bansal and Yaron (2004) the volatility of the price-dividend ratio generated by the long-run risk component is about .20, very close to what is obtained in this paper. Furthermore, in their case half of the volatility of the returns is generated by an additional dividend specific shock orthogonal to the consumption shocks. This is a common assumption in the asset pricing literature that is consistent with what is shown in the previous section: a good part of the aggregate dividends volatility is not related

²⁵See, for example, Tallarini (2000), Kaltenbrunner and Lochstoer (2006), and the CRRA case in Jerman (1998).

to productivity fluctuations, or, more in general, to macroeconomic uncertainty. This model, however, features complete markets, hence, it is possible to price a redundant asset paying the following cash-flows:

$$\Delta d_t^* = \Delta d_t + \epsilon_t^d, \quad \epsilon_t^d \sim iidN(0, \sigma_d),$$

where Δd_t are the dividends per share at the equilibrium, and ϵ_d is an exogenous and orthogonal cash-flow shock. By calibrating σ_d appropriately, it is possible to match the total volatility of US dividends growth. In this case, the returns volatility goes to about 10% per year, and the Sharpe-ratio becomes .5, a value closer to the data.

The risk-free rate has an annual standard deviation of .7%, a number included in the 95% confidence interval estimated in the data. This result is a success with respect to standard habit models, which are widely known to produce an excessively volatile risk-free rate.²⁶

Finally, in panel C of Table 5 I focus on co-movements, an important dimension the model is able to explore. Working with unfiltered consumption and investment time-series, we obtain a moderate contemporaneous correlation between these two variables. Indeed, while at business cycle frequencies consumption and investment are highly correlated, at lower frequencies they are not.

The model is perfectly able to reproduce this moderate correlation thanks to the presence of the long-run component. As seen before, while short-run shocks induce a perfect correlation between consumption and investment growth, long-run news forces consumption and investment to move in opposite directions. Overall, the correlation between these two variables implied by the model is still positive and perfectly in line with the empirical evidence. The same is true when we look at consumption and investment co-movements with output growth.

In the data the correlation between stock market returns, investment, and output growth is small. This is mostly induced by the fact that dividend growth is quite volatile but not very correlated with the macroeconomics variables just mentioned. The model, not surprisingly, over-predicts the correlation between stock market returns, investment growth, and output growth. In particular, what is at odds with the data is that capital income, dividend growth and returns all move in the same direction at exactly the same time after a short-run shock. Their contemporaneous correlation is higher than that observed in the data. In an exchange economy this problem is solved by introducing an idiosyncratic dividend-specific shock that reduces the correlation with macroeconomic fundamentals. In my production economy, however, this extra source of uncertainty for the dividends is not present.

The market returns are poorly correlated with consumption growth: in the data this correlation is about .6%, basically null, while in the model it is slightly negative, -10%. This result is extremely significative because it shows that the model produces high excess returns without altering the total correlation between consumption growth and stock market returns. This is

²⁶See, for example, Jerman (1998) and Boldrin, Christiano and Fisher (2001). Epstein and Zin (1989) preferences allow: (1) the conditional mean of the stochastic discount factor to have low volatility, and (2) the stochastic discount factor to have high conditional volatility.

one of the novelities of this kind of models.

3.4 Price-dividend ratio and exposure to productivity risk

One of the main reasons that the model is able to reproduce the properties of the stock returns is that it is able to generate sizeable time-variation in the price of capital thanks to the adjustment costs. Furthermore, as in the empirical analysis above, the model produces fluctuations of the price-dividend ratio that are positively related to long-run productivity shocks.

In Fig. 5 in the very last panels I plot the impulse response function of the price-dividend ratio with respect to both a short-run and a long-run shock. After a short-run shock, the price-dividend ratio actually falls and then mean-reverts geometrically toward its steady state. In response to a long-run shock, in contrast, the price-dividend ratio jumps up and then slowly mean-reverts, following a more complicated trajectory.

In order to better understand these dynamics, let us use the Campbell-Shiller approximation for the price-dividend ratio:

$$p_t - d_t = \bar{pd} + \sum_{j=0}^{\infty} \kappa_d^j E_t[\Delta d_{t+1+j}] - \sum_{j=0}^{\infty} \kappa_d^j E_t[r_{t+1+j}],$$

where κ_d is a positive approximation constant, and $|\kappa_d| < 1$. In the model described in this paper, the excess returns are almost constant over time. This allows me to write

$$p_t - d_t \approx \bar{pd}' + \sum_{j=0}^{\infty} \kappa_d^j E_t[\Delta d_{t+1+j}] - \sum_{j=0}^{\infty} \kappa_d^j E_t[r_{t+1+j}^f].$$

Solving for the risk-free rate it is then possible to obtain:

$$p_t - d_t \approx \bar{pd}'' + \sum_{j=0}^{\infty} \kappa_d^j E_t[\Delta d_{t+1+j}] - \sum_{j=0}^{\infty} \kappa_d^j \frac{1}{\Psi} E_t[\Delta c_{t+1+j}]$$

The price-dividend ratio is positively related to the conditional mean of future dividends growth and negatively related to the conditional mean of future consumption growth.

Under the benchmark calibration, the expected dividend growth induced by good long-run news is positive and is three times larger than that of consumption. Exactly as in Bansal and Yaron (2004), the positive long-run news about future dividends overcomes the movements in the discount factor related to the revisions of the expectations about future consumption growth. As a result, the price-dividend ratio increases.²⁷

The opposite is true for the short run-shocks. After the realization of positive short-run news to productivity, $\epsilon_a > 0$, the expected future dividend growth falls. This effect is very small, almost undetectable, but persistent. Moreover, the absolute value of this drop is five

²⁷The exposure of the expected dividends growth to long-run consumption risk is in reality time-varying. It is for this the reason that the price-dividend does not mean-revert geometrically.

times larger than that of the consumption growth conditional mean. Overall, the endogenous negative leverage of future dividend growth with respect to the short-run productivity shock generates the drop in the price-dividend ratio shown in the graph.

Why does future dividend growth become negative after idiosyncratic good news to productivity? The answer is once again related to the capital accumulation dynamics. When a short-run shock materializes, the agent responds immediately by increasing investment. After the shock, the agent begins to sustain consumption growth above its unconditional mean by consuming the extra units of the capital just accumulated. The capital-productivity ratio, K_t/A_{t-1} , slowly declines back to its steady state, and the same happens to the total dividend-productivity ratio, D_t/A_{t-1} . The negative growth of the dividend-productivity ratio in turn pushes the growth rate of total dividends below its unconditional average.

The relative sensitivity of the price-dividend ratio with respect to short- and long-run shocks is qualitatively consistent with what is seen in the empirical analysis. On impact, a 1% positive short-run shocks produces a 2% drop in the log price-dividend ratio. A 1% positive long-run shock, instead, lets the price-dividend ratio increase by 4%.

4 Inspecting the mechanism

In this section I perturb my benchmark calibration in order to study the role of the long-run component, the IES, and the adjustment costs. I show that the IES crucially determines the response of the quantities to long-run news. The presence of the long-run risk definitively helps to improve the model performance in several dimensions. Finally, I show that my adjustment costs produce better results than Jermann's. When the adjustment costs are not present and the price of capital is constant, the model is no longer able to replicate the stock market behavior even in the presence of long-run uncertainty.

4.1 The role of IES

The IES is a key parameter in this economy and in all the long-run risk models in general. Bansal and Yaron (2004) show that an IES greater than one is important to both maintain the unconditional mean of the interest rate at a low level and replicate the predictability of the excess returns in the presence of time-varying aggregate volatility. In this model I abstract from the presence of stochastic volatility, and for this reason my excess returns do not display any significant level of predictability. Still, the IES plays a crucial role in determining the response of endogenous quantities and, hence, the unconditional mean of the excess returns. To better show this point, I now solve the planner's problem for the case in which the IES is .9 keeping all the other parameters unchanged. In Fig. 6 I show the impulse responses related to this case.

By comparing Figs 6 and 5, we can see that in both cases the agent finds it optimal to respond to positive short-run shocks by increasing consumption and investment simultaneously. From a qualitative point of view, all the panels on the left hand side of the figure are the same.

The behavior of consumption and investment differs, however, with respect to the long-run component. As mentioned before, this is due to the interaction between the IES, the income effect, and the substitution effect produced by long-run news. It is now pertinent to observe the right-hand panels of Fig. 6 to be able to focus on the case in which a positive shock to the long-run productivity materializes. We know that the continuation value increases consistently. If the IES is low enough, current consumption and continuation value are complements: the income effect dominates, and the agent finds it optimal to increase consumption by reducing investment. This is exactly what we see in Fig. 6 and it is the opposite of what is shown in Fig. 5 with IES=2.

In order to compare these two different calibrations on a quantitative level, in Table 6 I report a list of relevant moments produced by simulating the model for the two values of the IES considered above. First of all, let us focus on the third and the last column of Table 6. When the IES=.9 there are three problems: (1) the volatility of consumption becomes too high, while investment volatility becomes too low; (2) the level of the risk-free rate is too high; and (3) the level of the market premium is 6 times smaller than that obtained with an IES of 2.

The origin of low risk premium becomes clear if we look once again at Fig. 6. Let us focus on the response of the market returns to a positive shock to long-run productivity. If the IES is too low, the returns fall below steady state in good times. This is generated by the fact that the demand of investment drops producing a substantial capital loss. Under this calibration, the market functions as an insurance device with respect to long-run news and for this reason it pays a much lower premium. Note that the total premium remains positive because the short-run shock is still a source of risk for the market returns. In this framework, asset prices clearly suggest that we should adopt an IES higher than the one used in the standard RBC literature. So far, the analysis shows that a higher IES does not imply any counterfactual implication for the co-movements of the macro variables.

4.2 The role of Productivity Long-Run Risk

What do we gain from introducing productivity long-run risk in our macro models? Equivalently, what do we lose if we neglect long-run uncertainty? To answer these questions, in Table 6 I also show the model performance when productivity follows a random walk with constant drift. I denote this case as "NO LRR". Let us focus on the last two columns of the table. Neglecting long-run productivity uncertainty produces the following problems: (1) both consumption and investment become too smooth and their contemporaneous correlation becomes too high; (2) the equity premium goes from 3% under the benchmark calibration to just 1.5%, despite the fact that the correlation between consumption growth and the excess returns is much higher; (3) the risk-free rate is almost three times higher than before and its volatility becomes too low.

4.3 The role of adjustment costs

As in Boldrin, Christiano and Fisher (2001), the economy proposed in this paper can also be decentralized assuming the existence of two sectors. One sector produces consumption goods, the other one investment:

$$\begin{aligned} K_{c,t}^\alpha (A_t n_{c,t})^{1-\alpha} &= C_t, & K_{i,t}^\alpha (A_t n_{i,t})^{1-\alpha} &= I_t; \\ n_{c,t} + n_{i,t} &= \bar{n}, & K_{c,t} + K_{i,t} &= K_t^{tot}. \end{aligned}$$

Here $n_{c,t}$ and $n_{i,t}$ denote hours worked in the consumption and investment sectors, respectively. Similarly, $K_{c,t}$ and $K_{i,t}$ refer to the allocation of capital across the two sectors. Total capital, K^{tot} , follows the dynamics described below:

$$\begin{aligned} K_{t+1}^{tot} &= [1 - \delta_k - G(K_{i,t}/K^{tot})] K_t^{tot} + I_t, \\ G\left(\frac{K_{i,t}}{K_t^{tot}}\right) &= \left[a_1 \left| \frac{K_{i,t}}{K_t^{tot}} - \frac{\bar{K}_i}{K_t^{tot}} \right| \right]^\tau. \end{aligned}$$

Given these assumptions, total output, Y_t , can be expressed as:

$$Y_t \equiv C_t + I_t = (K_t^{tot})^\alpha (A_t \bar{n})^{1-\alpha}.$$

In this economy, reallocating capital across sectors is costly. Since it is possible to show that at the equilibrium $\frac{I_t}{Y_t} = \frac{K_{i,t}}{K_t^{tot}} \quad \forall t$, the adjustment costs can be written as a function of the investment-output ratio (I/Y) as in (10). Boldrin, Christiano and Fisher (2001) assume that it takes one period to change the relative size of the two sectors, and for this reason I/Y is a predetermined state variable that generates fluctuation in asset prices. In my economy, instead, the agent can alter the composition of output in every period just by paying a small adjustment cost. Since the I/Y ratio is not an additional state variable, the numerical solution of the model remains simple. Prices, however, are still driven by the I/Y ratio.

Before proceeding with the sensitivity analysis, in figure 7 I show the scatter plot of the log price-dividends ratio and the investment-output ratio in the US, together with a simple OLS regression line. In the 1949-2003 sample, the statistical relationship between these two variables has been positive and significant.²⁸ This basic analysis suggests that the adjustment costs proposed in this paper could be empirically plausible.

In Table 7 I show the results obtained by employing the standard capital adjustment costs (fifth column) and the benchmark adjustment costs (third column). In order to make the results really comparable, I choose the Jermann's adjustment costs elasticity, ξ , to deliver exactly the same equity premium obtained under the benchmark calibration. Thanks to the presence of long-run risk I am able to impose $\xi = 3$, a value less extreme than that used by Jermann

²⁸The same is true when we look at the US measured q . This is not surprising given the high correlation between the measured q and the p-d ratio.

(1998).²⁹ Despite the milder amount of adjustment costs, the model produces a substantially smooth investment time-series and a much more volatile consumption profile. Furthermore, the correlations between consumption growth, investment growth and excess returns tend to become too big with respect to the data. The output composition costs, instead, reproduce all these factors significantly better.

In order to understand why this formulation works better, notice that the level of investment $I_t^0 : G(I_t^0/Y_t) = 0$ can be written as $I_t^0 = \frac{\bar{I}}{\bar{Y}}Y_t$. Short-run shocks move I_t^0 exactly in the direction in which the agent wants to adjust current investment, I_t . Long-run shocks, instead, do not move I_t^0 since they do not affect current output. For this reason, these costs are basically mild with short-run shocks (so that the economy behaves almost like one without costs), but more binding when a long-run shock realizes. This is very useful since it allows me to obtain volatility in investment through the short-run shock and volatility in prices through the long-run shocks.

Finally, let us compare the results obtained under the benchmark calibration (second column) with those produced by an economy in which there are no adjustment costs (third column). When the adjustment costs are not present, the agent finds it more convenient to intensively and persistently vary capital subsequent to both short-run and long-run shocks. As documented in Table 7, the absence of any investment friction produces the following problems: (1) both investment and consumption growth become too volatile and negatively correlated; (2) investment and consumption become perfectly negatively correlated because of the dominance of the long-run shock; (3) the equity premium goes down by half of its value under the benchmark calibration; and (4) the risk free rate is too high.

The interaction between the IES and the elasticity of the adjustment costs function is crucial to an understanding of the negative co-movements between consumption and investment. When the supply of new capital is flat and the IES is high enough, the substitution effect dominates the income effect also in the short run. When the adjustment of capital is costless, the opportunity cost of consumption increases significantly, even after short-run shocks. If the agent has a high elasticity of substitution, she will find it optimal to decrease consumption and also invest more after the realization of a transitory positive shock to productivity growth. Overall, this implies a counterfactual negative correlation between consumption and investment as well as between consumption and returns.³⁰

5 Conclusion

This study has examined the intertemporal distribution of productivity risk. Focusing on post-war US data, I have shown that the conditional mean of productivity growth is time-varying and extremely persistent. This generates uncertainty about the long-run perspectives of economic growth and strongly affects asset prices. I developed a production-based asset pricing model

²⁹In Jermann (1998) $\xi = .23$.

³⁰See the Technical Appendix.

featuring long-run uncertainty, convex adjustment costs related to the output composition, and recursive preferences à la Epstein-Zin. The model accounts for several key features of both prices and macroeconomic quantities, including consumption, investment, and output. I have also provided a detailed examination of the role of the intertemporal elasticity of substitution, the long-run productivity risk, and adjustment costs in this type of economy.

There are a number of directions in which this work could be fruitfully extended. First, in this paper I assume a constant supply of labor in order to keep the analysis as simple as possible. However, long-run labor fluctuations can have an important impact on human wealth returns (see, for example, Lustig and Van Nieuwerburgh (2006)) and should be taken into account in future research. In particular, it would be especially useful to study the implications of the model for the relationship between labor income, aggregate variables, and capital returns.

Second, as we have seen, the model predicts excessively smooth returns. It would be valuable to consider other endogenous factor that can enrich the dynamics of the dividends and better match the volatility of the returns. Croce, Kung and Schmidt (2009), for example, explore the role of endogenous financial leverage in a production economy with long-run uncertainty on the tax rate.

Finally, it would also be interesting to study the implications of time-varying volatility for productivity growth. This could help us to better understand the dynamics of aggregate risk premiums.

Appendix A. Data.

Data about real annual consumption and investment are from the Bureau of Economic Analysis (BEA). I compute output as the sum of consumption and investment. I exclude government expenditures and net export. All data are per-capita.

I measure inflation using the CPI index, from 1949 to 2003. The real risk-free rate is measured by the 3-month T-Bill return minus the realized inflation, from 1949 to 2003.

Data on market returns, annual price-dividend ratio, and dividends are obtained from CRSP. Annual dividends are obtained by time aggregating monthly ones. Nominal dividends are turned into real ones using the CPI index.

The time-series for annual productivity is from the Bureau of Labor Statistics; the sample spans 1949–2003. I use a multifactor productivity index that takes into account capital accumulation. In particular, the index adopted measures the value-added output per combined unit of labor and capital input in private business and private nonfarm business (as Kaltenbrunner and Lochstoer (2006)).

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TABLE 1
PREDICTABILITY IN PRODUCTIVITY GROWTH

$$\Delta a_{t+1} = \mu(1 - \rho) + \rho\Delta a_t - b\sigma_v v_{a,t} + \sigma_v v_{a,t+1}$$

ρ	b	$R^2_{\Delta a}$	LR	ρ_x	R^2_x	ρ_σ	b_σ
No GARCH(1,1)							
.965	1.191	.250	10.801	.835	.720	-	-
(.061)	(.149)	-	-	(.073)	-	-	-
[.000]	[.000]	-	[.004]	[.000]	-	-	-
With GARCH(1,1)							
.892	.965	.106	7.74	.894	.954	.520	.420
(.043)	(.014)	-	-	(.073)	-	(.156)	(.142)
[.000]	[.000]	-	[.10]	[.000]	-	[.003]	[.004]

In the first three columns I report, respectively, the estimates of the AR root, the MA root, and the R^2 of the regression for Δa_{t+1} . Numbers in parentheses are Newey-West adjusted standard errors, while numbers in square brackets are p -values. In the fourth column I report the likelihood ratio of the estimated model versus the following model:

$$\Delta a_{t+1} = \mu + \sigma\epsilon_{a,t+1}.$$

In columns 5 and 6 I report the estimated AR coefficient and the R^2 of the following regression:

$$x_t = \mu(1 - \rho_x) + \rho_x x_{t-1} + \epsilon_{x,t}$$

where x_t is the ARMA(1,1) forecast of Δa_{t+1} . In the last two columns I report the estimates of the AR and the MA root of the conditional volatility that is assumed to follow a GARCH(1,1). In the lower portion of the table I report the *joint* estimates of both the ARMA(1,1) and the GARCH(1,1) coefficients. Data are annual (1948–2003).

TABLE 2
LONG-RUN PRODUCTIVITY RISK AND P/D RATIO NEWS

$\epsilon_{pd,t} = \beta_0 + \beta_1 v_{x,t} + \epsilon_t$		
β_1	F -stat	$R^2_{\Delta a}$
No GARCH(1,1)		
11.107	6.426	.112
(5.222)	-	-
[.038]	[.014]	-
With GARCH(1,1)		
22.967	3.581	.065
(12.135)	-	-
[.064]	[.064]	-

The variables ϵ_x and ϵ_{pd} are the residuals of the following regressions:

$$\begin{aligned}
 p_t - d_t &= \beta_{0,pd} - \rho_{pd}(p_{t-1} - d_{t-1}) + \epsilon_{pd,t} \\
 x_{v,t} &= \beta_{0,x} - \rho_x x_{v,t-1} + v_{x,t}
 \end{aligned}$$

where x_t is the ARMA(1,1) forecast of the productivity growth rate, Δa_{t+1} . In the upper portion of the table I report the OLS estimates obtained using as the measure of productivity long-run risk the forecast provided by the ARMA(1,1) model not corrected for GARCH effects. In the lower portion of the table I report the forecast obtained by controlling for GARCH effects. Numbers in parentheses are Newey-West adjusted standard errors, while numbers in square brackets are p -values. Data are annual; the adjusted sample is for the years 1951–2003.

TABLE 3
PREDICTABILITY IN PRODUCTIVITY GROWTH (II)

Bivariate Approach								
ρ	σ_x	σ	ρ_{xa}	$R_{\Delta a}^2$	ρ_{pd}	α_1	α_2	J-Stat
.935	.004	.022	.514	.085	.928	48	-5.1	4.07
(.108)	(.09%)	(.003)	(.397)	-	(.059)	(19.35)	(1.94)	-
[.000]	[.000]	[.000]	[.195]	-	[.000]	[.012]	[.008]	[.130]

In the first row I report point estimates obtained with a Continuous-GMM estimator applied to the following system of equations:

$$\begin{aligned}
 \Delta a_{t+1} &= x_t + \sigma \epsilon_{a,t+1} \\
 x_t &= \rho x_{t-1} + \sigma_x \epsilon_{x,t} \\
 pd_{t+1} &= \rho_{pd} pd_t + \alpha_1 \sigma_x \epsilon_{x,t+1} + \alpha_2 \sigma \epsilon_{a,t+1} \\
 \begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} &\sim iidN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right).
 \end{aligned}$$

Numbers in parentheses are Newey-West adjusted standard errors, while numbers in square brackets are p -values. The value $R_{\Delta a}^2$ is computed as the ratio of the variances of the fitted long-run component and of total productivity growth. Data are annual (1948–2003).

TABLE 4
MACROECONOMIC GROWTH IN THE LONG- AND SHORT-RUN

Bivariate Approach						
Dependent Variable	Regressors					R^2
	$\epsilon_{a,t}$	x_{t-1}	$\epsilon_{a,t}-\epsilon_{a,t-1}$	$x_{t-1}-x_{t-2}$	$\epsilon_{a,t-1}$	
(1) Δc_t	.255 (.053) [.000]	.451 (.211) [.037]				.419
(2) Δi_t	1.56 (.114) [.000]	1.749 (.853) [.045]				.445
(3) Δy_t	.558 (.065) [.000]	.812 (.326) [.016]				.438
(4) Δd_t	.97 (.217) [.000]	1.383 (.703) [.054]				.290
(5) Δn_t			.060 (.061) [.329]	3.668 (.512) [.000]		.461
(6) Δn_t	.258 (.083) [.003]			2.210 (.817) [.009]	.232 (.112) [.044]	.530

I report the result of OLS projections of the dependent variables listed on the first column onto the selected regressor (col. 2-6). The variables $\epsilon_{a,t}$, x_t , and $\epsilon_{x,t}$ have been computed using the point estimates reported in table 3. Numbers in parentheses are Newey-West adjusted standard errors, while numbers in square brackets are p -values. Data are annual (1948–2003).

TABLE 5
 PRODUCTION ECONOMY: BENCHMARK CALIBRATION

Panel A: Calibration										
μ	σ	σ_x	ρ	δ_k	α	τ	a_1	δ^{12}	γ	Ψ
.165%	.8%	5.5% σ	.98	.5%	.33	1.9	.07	.98	30	2

Panel B: General Statistics				
	Mean		St. Dev.	
	Model	Data	Model	Data
C_{t+1}/C_t	2.0	1.9 (.2)	1.5	1.2 (.1)
I_{t+1}/I_t	2.0	2.5 (.7)	5.1	6.7 (.6)
I_t/Y_t	29	26 (.8)	2.8	2.9 (.6)
$r_{t+1}^d - r_t^f$	3.0	6.0 (2.0)	1.6	16.0 (1.8)
r_t^f	1.1	.90 (.24)	.7	1.0 (.17)
$p_t - d_t$	3.59	3.46 (.11)	.18	.40 (.08)

Panel C: Cross-Correlations						
	DATA			MODEL		
	ΔY_t	ΔI_t	ΔC_t	ΔY_t	ΔI_t	ΔC_t
ΔI_t	95 (0.8)			ΔI_t	92	
ΔC_t	85 (3.3)	68 (6.2)		ΔC_t	85	61
r_t^d	1.2 (11)	-.01 (.12)	.06 (13)	r_t^d	19	43 -10

Panel A shows the benchmark monthly calibration. In panels B and C, all the statistics are multiplied by 100, except those ones for the log price-dividends ratio, $p_t - d_t$. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to an annual frequency. Data are annual (1949–2003), numbers in parenthesis are Newey-West adjusted standard errors obtained through a continuous GMM estimator; more details can be found in Appendix A. In this table $\bar{l} = .82$.

TABLE 6
THE ROLE OF IES (Ψ) AND LRR (x)

Panel A: Calibration									
μ	σ	σ_x	ρ	δ_k	α	τ	a_1	δ^{12}	γ
.165%	.8%	5.5% σ	.98	.5%	.33	1.9	.07	.98	30
Panel B: Statistics									
	$\Psi = .9$	$\Psi = .9$	$\Psi = 2$	$\Psi = 2$					
	NO LRR	WITH LRR	NO LRR	WITH LRR					
std[C_{t+1}/C_t]	01.08	01.70	00.80	01.50					
std[I_{t+1}/I_t]	02.94	04.20	04.00	05.10					
$\rho_{\Delta c, \Delta i}$	96.00	57.00	80.00	61.00					
$\rho_{\Delta c, r^d - r^f}$	60.00	-3.81	30.00	-10.0					
$E[r_{t+1}^d - r_t^f]$	01.10	0.51	01.50	03.00					
$E[r_t^f]$	03.70	02.02	02.70	01.10					
std[$r_{t+1}^d - r_t^f$]	00.65	01.10	01.10	01.60					
std[r_t^f]	00.35	00.94	00.22	00.70					
std[$p_t - d_t$]	06.43	25.01	10.67	18.00					
ACF ₁ [$r_{t+1}^d - r_t^f$]	06.00	-7.00	00.00	15.00					
ACF ₁ [r_t^f]	90.00	92.00	88.00	92.00					
ACF ₁ [$p_t - d_t$]	90.00	98.00	90.00	97.00					

Panel A shows the benchmark monthly calibration. In panel B, all the statistics are annualized and multiplied by 100. Data are annual (1949–2003); more details can be found in Appendix A. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to an annual frequency. In this table, $\bar{l} = .82$.

TABLE 7
THE ROLE OF THE ADJUSTMENT COSTS

Panel A: Calibration common parameters								
μ	σ	σ_x	ρ	δ_k	α	δ^{12}	γ	Ψ
.165%	.8%	5.5% σ	.98	.5%	.33	.98	30	2
Panel B: Statistics								
	DATA	Benchmark Model	No Adj. Costs	Jermann Adj. Cost				
std[C_{t+1}/C_t]	01.20 (.10)	01.50	01.80	01.60				
std[I_{t+1}/I_t]	06.70 (.60)	05.10	07.83	03.70				
$\rho_{\Delta c, \Delta i}$	68.00 (6.2)	61.00	-14.0	80.00				
$\rho_{\Delta c, r^d - r^f}$	00.06 (13)	-10.0	-90.0	60.00				
E[$r_{t+1}^d - r_t^f$]	06.00 (2.0)	03.00	01.50	03.00				
E[r_t^f]	00.90 (.24)	01.10	01.82	01.10				
std[$r_{t+1}^d - r_t^f$]	16.00 (1.8)	01.60	00.70	02.00				

Panel A shows the calibration of the unchanged parameters. In panel B, all the statistics are annualized and multiplied by 100. Data are annual (1949–2003); more details can be found in Appendix A. The entries for the models are based on 500 simulations each with 600 monthly observations that are time-aggregated to an annual frequency. In this table, $\bar{l} = .82$. In the last column, the adjustment costs are as in Jermann (1998), with the elasticity ξ equal to 3.

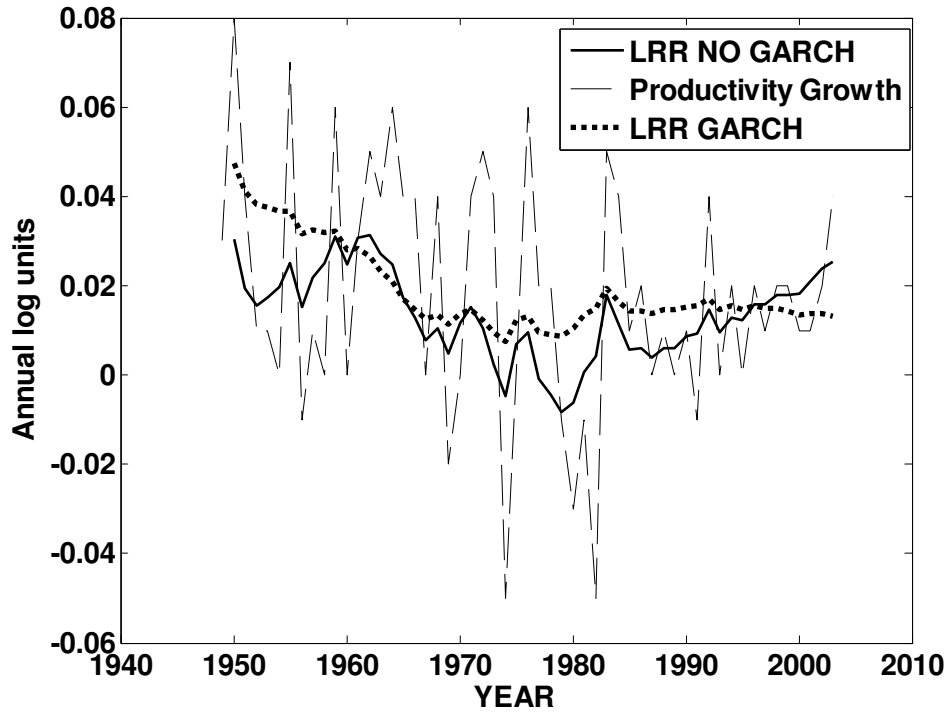


FIG. 1 – THE LONG-RUN PRODUCTIVITY RISK

The dashed line shows the annual growth rates of aggregate US productivity (source: BLS; sample: 1948–2003). The solid thick line shows the fit from an ARMA(1,1) model with homoskedastic innovations. The dotted line shows the estimated conditional mean of the productivity growth rate obtained by estimating the following ARMA(1,1)-GARCH(1,1) model:

$$\begin{aligned}
 \Delta a_{t+1} &= \mu(1 - \rho) + \rho\Delta a_t - b\sigma_t\epsilon_{a,t} + \sigma_{t+1}\epsilon_{a,t+1} \\
 \sigma_{t+1}^2 &= \kappa + \rho_\sigma\sigma_t^2 + b_\sigma\sigma_t^2\epsilon_{a,t}^2 \\
 \epsilon_{a,t+1} &\sim iidN(0, 1),
 \end{aligned}$$

where Δa_{t+1} denotes the log growth rate of productivity between time t and $t + 1$. All the parameter estimates are reported in Table 1.

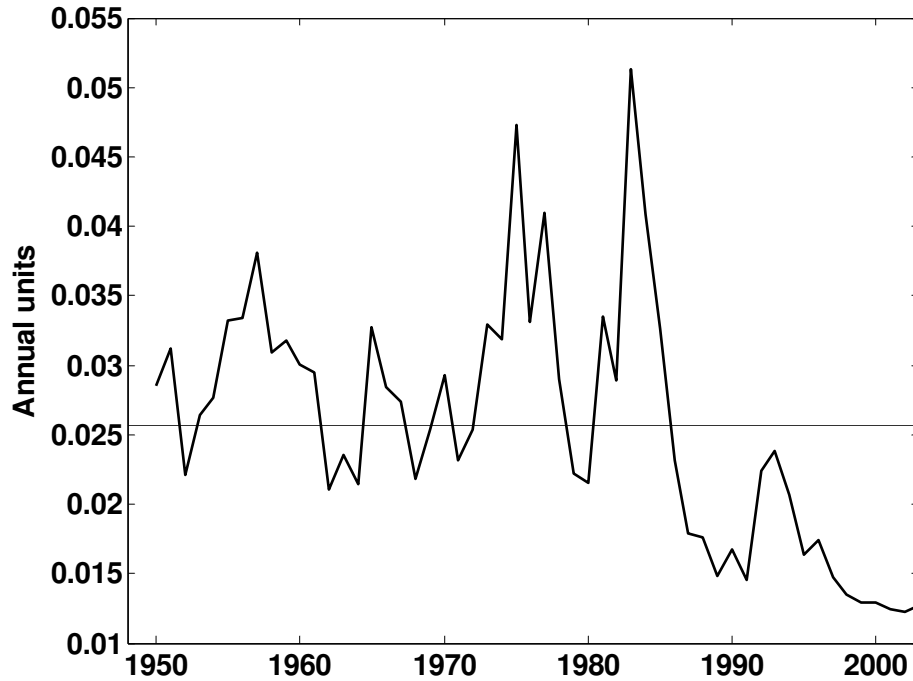


FIG. 2 – VOLATILITY OF PRODUCTIVITY GROWTH RATES

The thick line shows the fitted conditional volatility, $\sqrt{\widehat{\sigma}_t^2}$, of the annual growth rate of aggregate US productivity (source: BLS; sample: 1948–2003). The model employed is the following:

$$\begin{aligned}\Delta a_{t+1} &= \mu(1 - \rho) + \rho\Delta a_t - b\sigma_t\epsilon_{a,t} + \sigma_{t+1}\epsilon_{a,t+1} \\ \sigma_{t+1}^2 &= \kappa + \rho_\sigma\sigma_t^2 + b_\sigma\sigma_t^2\epsilon_{a,t}^2 \\ \epsilon_{a,t+1} &\sim iidN(0, 1),\end{aligned}$$

where Δa_{t+1} denotes the log growth rate of productivity between time t and $t + 1$. The thin flat line shows the unconditional volatility of the productivity growth rate in the same sample. All the parameter estimates are reported in Table 1.

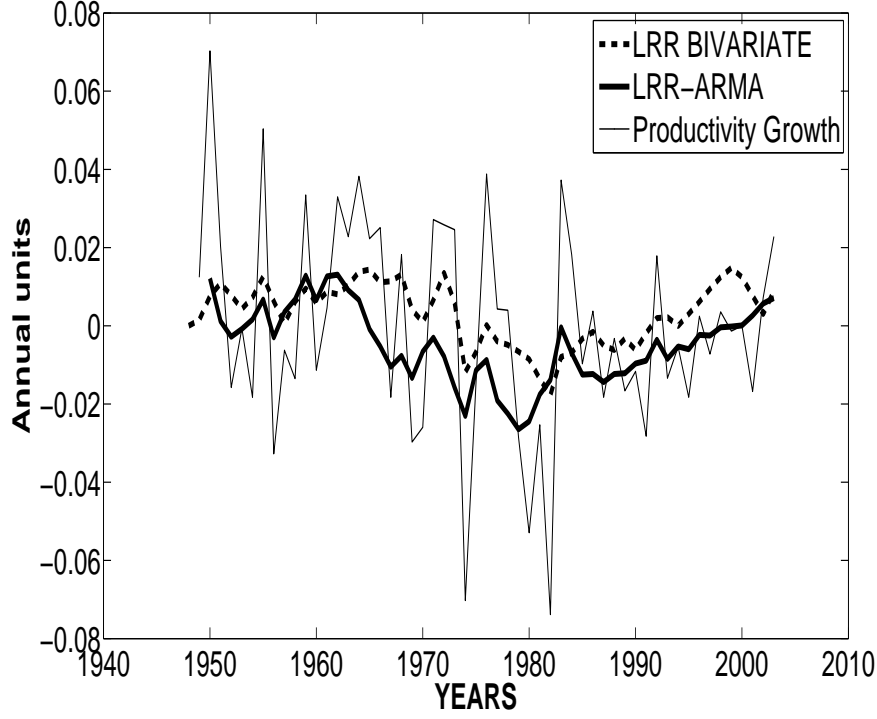


FIG. 3 – LONG-RUN PRODUCTIVITY NEWS (II)

The thin solid line shows the demeaned annual growth rate of aggregate US productivity (source: BLS; sample: 1948–2003). The solid thick line shows the fit from an ARMA(1,1) model with homoskedastic innovations. The dotted line shows the estimated conditional mean of the productivity growth rate obtained by estimating the following model:

$$\begin{aligned}
 \Delta a_{t+1} &= x_t + \sigma \epsilon_{a,t+1} \\
 x_t &= \rho x_{t-1} + \sigma_x \epsilon_{x,t} \\
 pd_{t+1} &= \rho_{pd} pd_t + \alpha_1 \sigma_x \epsilon_{x,t+1} + \alpha_2 \sigma \epsilon_{a,t+1} \\
 \begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} &\sim iidN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right).
 \end{aligned}$$

where Δa_{t+1} denotes the log growth rate of productivity between time t and $t + 1$ and pd_t denotes the demeaned annual log-price-dividends ratio for the US (source: CRPS). All the parameter estimates are reported in Table 3.

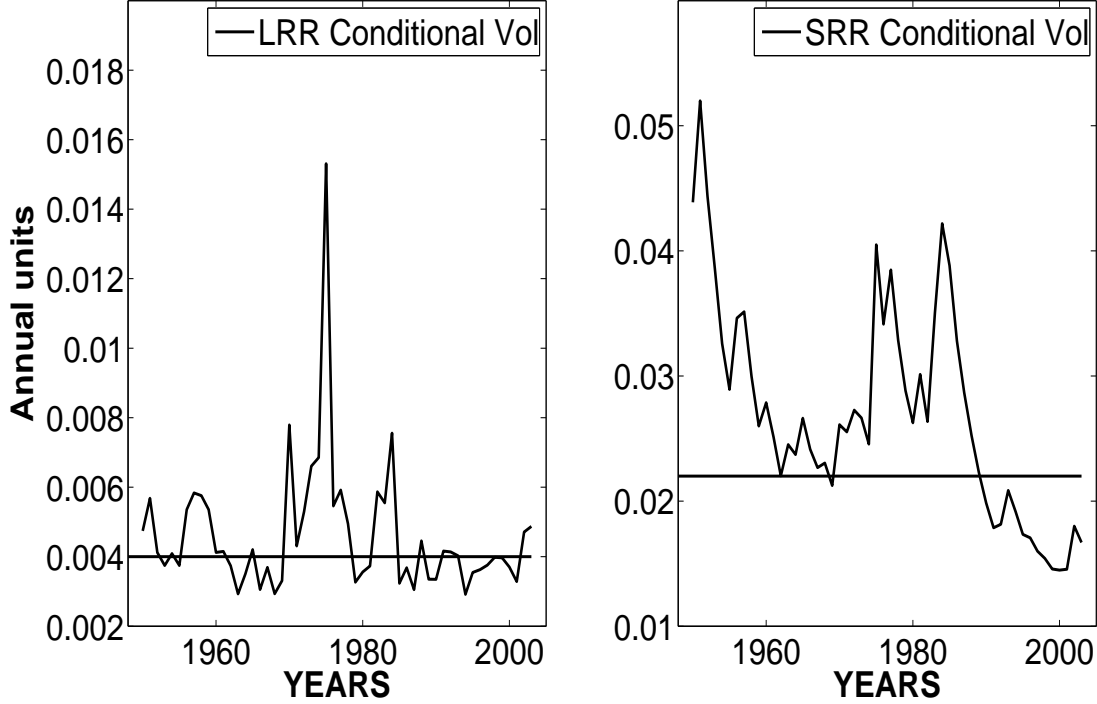


FIG. 4 – LONG-RUN PRODUCTIVITY NEWS (II)

This figure shows both the unconditional and the conditional volatility of both the long-run shocks (left panel) and the short-run shocks (right panel). The time-varying volatilities are obtained by estimating a GARCH(1,1) on the fitted shocks $\hat{\epsilon}_{x,t}$ and an ARCH(1) on $\hat{\epsilon}_{a,t}$ (these shocks are the output of the estimation documented in table 3). Below I report the estimates for the AR root, $\rho_{\sigma,i}$, and the MA coefficient, $b_{\sigma,i}$, of the volatility process for the corresponding shock $\hat{\epsilon}_{i,t}$, $i \in \{x, a\}$:

	ρ_{σ}	b_{σ}
SRR	.637	-
	(.392)	-
	[.105]	-
LRR	.240	.678
	(.134)	(.137)
	[.070]	[.000]

Numbers in parentheses are standard errors, while numbers in square brackets are p -values.

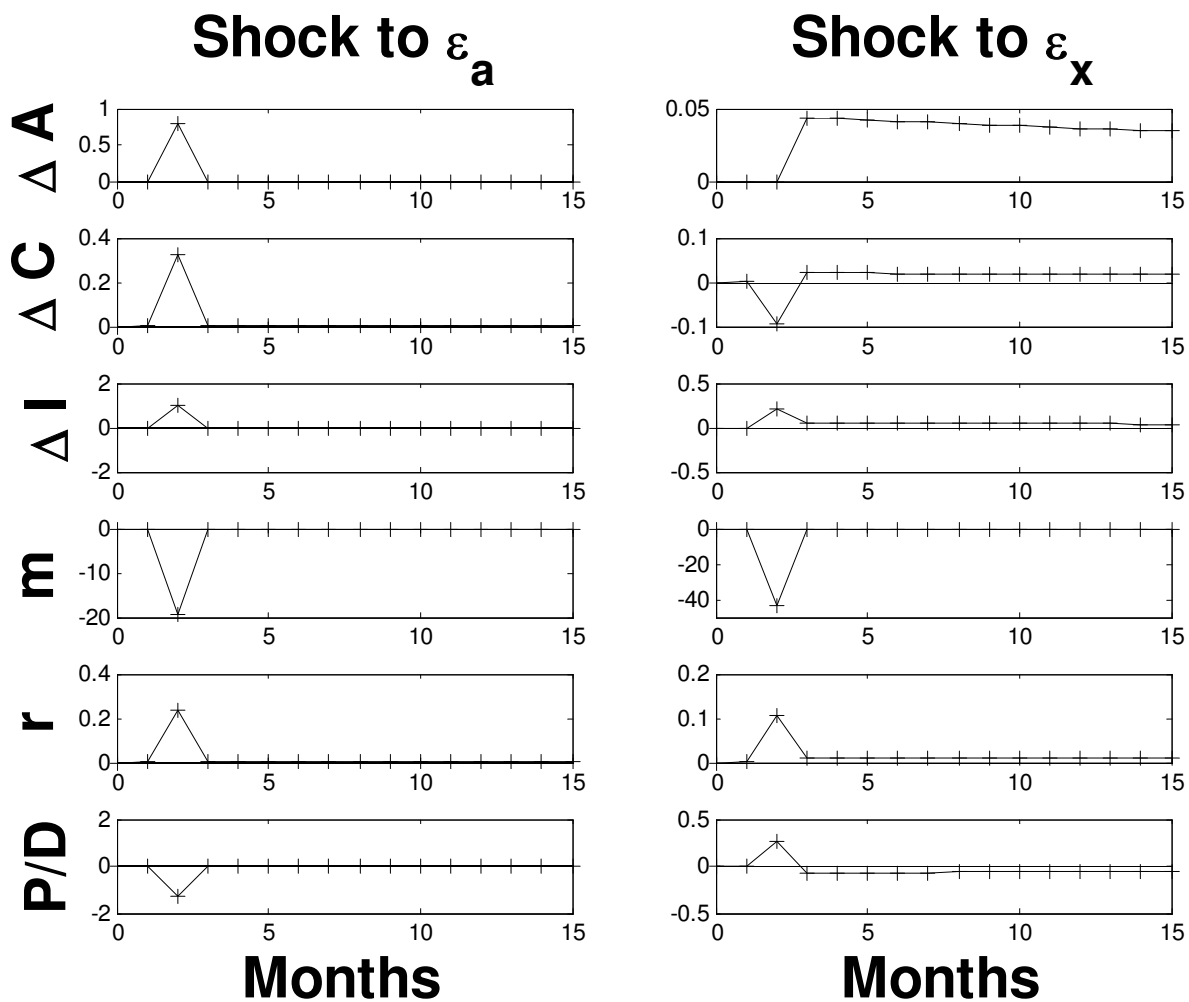


FIG. 5 – IMPULSE RESPONSE FUNCTIONS (IES = 2)

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 5. The policy functions are computed numerically. Returns are not levered.

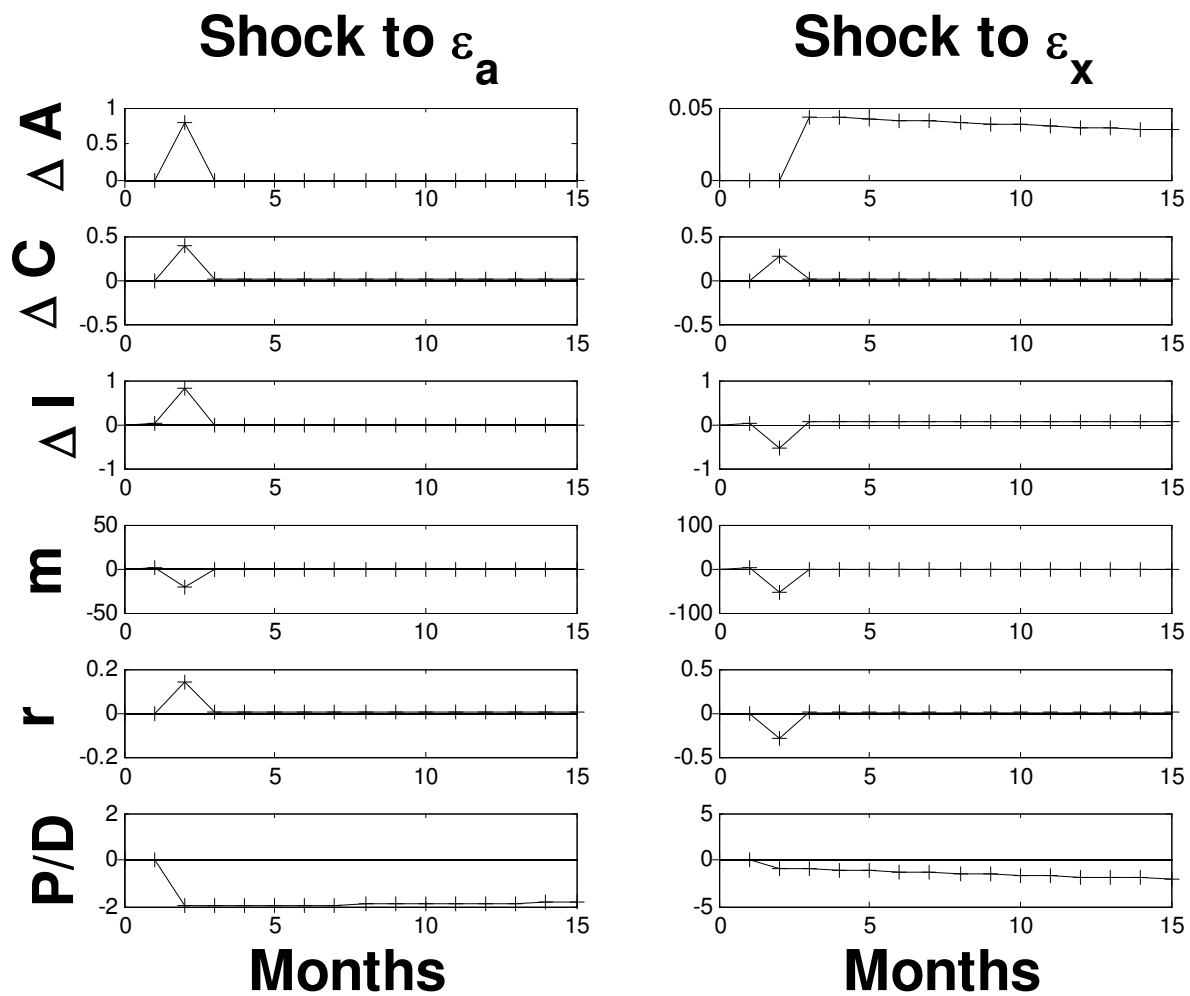


FIG. 6 – IMPULSE RESPONSE FUNCTIONS (IES = .9)

This figure shows monthly log-deviations from the steady state. Units are multiplied by 100. All the parameters are calibrated to the values reported in Table 6. The policy functions are computed numerically. Returns are not levered.

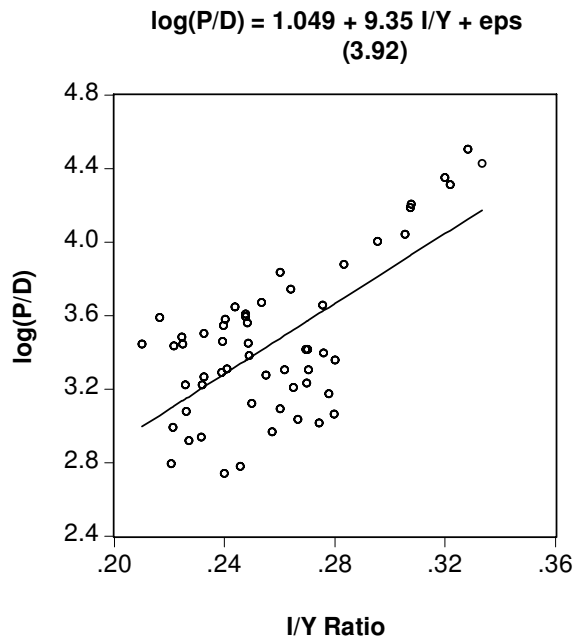


FIG. 7 – P/D AND I/Y RATIOS IN THE US

This figure shows the scatter plot for the US price-dividend ratio and the investment-output ratio. Data are annual and range from 1949 to 2003. The number in parenthesis is a t-stat. Standard errors are Newey-West adjusted.